Delaying Room Reflections: Constraints on Room Size and Loudspeaker Placement

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1 Introduction

When loudspeakers interact with room boundaries and other acoustically reflective surfaces, the delay between direct and reflected sound waves affects not only the perceived timbre, but also the degree to which believable phantom stereo images are created. If reflected sound waves arrive too soon after the direct sound, they generate spurious directional cues that can spoil the stereo-imaging illusion.

The precedence effect offers a remedy for the effect of room reflections on stereo imaging: to the extent that the reflected sound is a sufficiently delayed copy of the direct sound, with similar spectral and temporal content, the auditory system will take directional cues only from the direct sound [1, ch. 6]. To make this work we need loudspeakers with frequency-independent polar response (i.e. "constant directivity") and we need to place them far enough from reflecting surfaces.

There is some uncertainty about the minimum delay needed for precedence to take full effect, since it is both signal- and level-dependent. An early experiment by Haas [2, 3] suggests 1 ms is just sufficient. Later work [4, 5] found that delays of about 10 ms give the highest reflection level threshold for barely detectable image shifts. Figure 6.16 of Toole [1] would suggest that in a "typical" room, a delay of at least 5 ms will put reflection levels below the threshold where they can be perceived as a second image. Linkwitz [6] recommends at least 6 ms delay, and this appears to be a sensible target.

To delay the arrival of the reflected sound at the listening position by 6 ms relative to the direct sound, the path length for the reflected sound must be at least $(343\,\mathrm{m/s}) \times (6\,\mathrm{ms}) \approx 2.1\,\mathrm{m}$ longer than the direct path from loudspeaker to listener. This path length difference can be achieved only if the room is sufficiently large. One wonders: for a given arrangement of loudspeakers and listener, what is the smallest rectangular room where a 6 ms delay in first-reflection arrivals can be achieved?

2 Lateral First-Reflection Geometry

Consider a loudspeaker and listener as in Figure 1, which illustrates the geometry of a first-order lateral reflection from a wall. The listener is at A, with the loudspeaker at B. Point C is the image of the loudspeaker in the wall. The reflected sound path BOA has the same length as the hypotenuse AC, so we have

$$\begin{aligned} \text{direct path length} &= \overline{AB} = r \\ \text{reflected path length} &= \overline{AC} = \sqrt{(2x-u)^2 + v^2}. \end{aligned}$$

Here u, v are the components of the loudspeaker-listener displacement measured perpendicular and parallel to the reflecting wall, respectively; x is the distance from the listener to the reflecting wall.

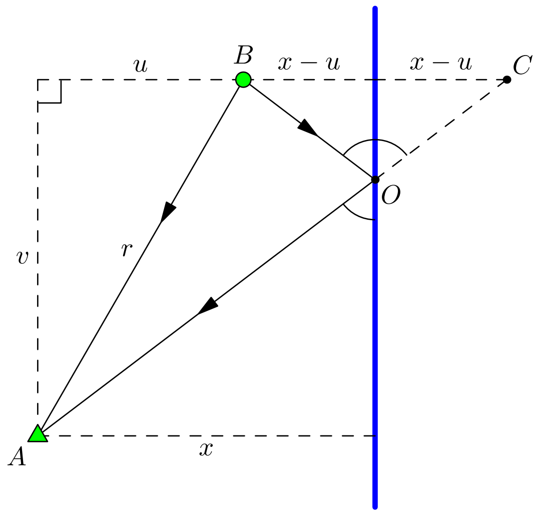


Figure 1: Direct and reflected sound paths from a loudspeaker (green circle) to a listener (green triangle). Point C is the reflected image of the loudspeaker in the wall. The hypotenuse $\overline{AC}$ gives the reflected path length $\overline{BOA}$.

A little simplification gives

$$\begin{aligned}\overline{AC} &= \sqrt{4x^2 - 4ux + u^2 + v^2} \\ &= \sqrt{4x^2 - 4ux + r^2}.\end{aligned}$$

The difference in path lengths determines the difference in arrival times at the listening position. Thus, with c denoting the speed of sound in air, the arrival time difference T is given by

$$\begin{aligned} T = \frac{\overline{AC} - \overline{AB}}{c} &\implies cT = \sqrt{4x^2 - 4ux + r^2} - r \qquad (1) \\ &\implies 4x^2 - 4ux + r^2 = (r + cT)^2 \\ &\implies x^2 - ux - \tfrac{1}{4}\left(2rct + c^2T^2\right) = 0. \end{aligned}$$

The quadratic formula then gives

$$x = \tfrac{1}{2}\left(u + \sqrt{u^2 + 2rcT + c^2T^2}\right) \qquad (2)$$

as the distance the listener must be from the wall, to achieve a given arrival time delay T. (The other, spurious root gives $x < 0$.)

3 Equilateral Placement: An Example

Consider the typical arrangement shown in Figure 2, where the listener and a stereo pair of loudspeakers form an equilateral triangle that shares an axis of symmetry with the room. In the following example we take the stereo separation r to be $8\,\text{ft} \approx 2.44\,\text{m}$. For reflections from both the

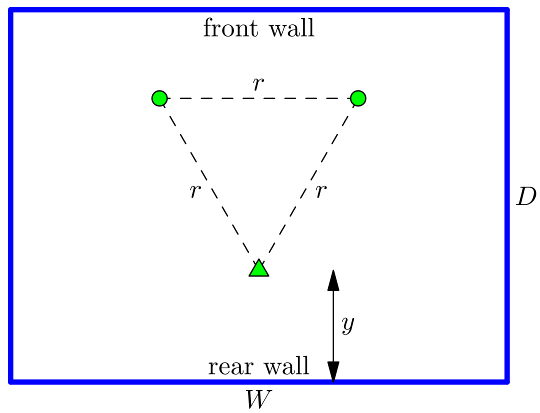


Figure 2: Typical symmetric equilateral arrangement of a listener (green triangle) and stereo pair of loudspeakers (green circles) in a rectangular room.

left and right side walls we have (see Figure 1)

$$u = r\cos(60°) = \tfrac{1}{2}r \approx 1.22\,\mathrm{m}$$

and (with $c = 343\,\mathrm{m/s}$) equation (2) gives $x \approx 2.59\,\mathrm{m}$ as the distance from listener to side wall to achieve a delay of $T = 6\,\mathrm{ms}$. Thus

$$2x \approx 5.19\,\mathrm{m} \approx 17.0\,\mathrm{ft}$$

is the minimum room width for 6 ms delay of side wall reflections. This is somewhat greater than the 4.5 m minimum that Linkwitz gives in [6]. The distance from the loudspeaker to the side wall is $x - u \approx 1.4\,\mathrm{m} \approx 4.5\,\mathrm{ft}$.

For the reflection off the front wall we have

$$u = r\sin(60°) = \frac{\sqrt{3}}{2}r \approx 2.11\,\mathrm{m}$$

in equation (2), giving $x \approx 3.22\,\mathrm{m}$ as the distance from listener to front wall to achieve 6 ms delay. For the rear wall reflection the quantities are the same, except the loudspeaker and listener locations in Figure 1 are reversed and the minimum distance from listener to rear wall is $x - u \approx 1.11\,\mathrm{m}$. Thus the minimum room depth for 6 ms delay of both front and rear wall reflections is

$$x + (x - u) = 2x - u \approx 4.33\,\mathrm{m} \approx 14.2\,\mathrm{ft},$$

quite a bit less than 6 m minimum that Linkwitz suggests. By symmetry the loudspeakers also are at $x - u \approx 1.1\,\mathrm{m} \approx 3.6\,\mathrm{ft}$ from the front wall.

Figure 3 illustrates this smallest room for which a 6 ms reflection delay can be achieved with 8 ft equilateral placement, together with the necessary placement of the listener and loudspeakers with respect to the room. Superimposed on the room is a contour plot of the first reflection delay time (minimum of all four delays from equation (1)) as a function of loudspeaker placement, for the fixed listening position shown. Loudspeakers must be placed in the shaded region to achieve at least

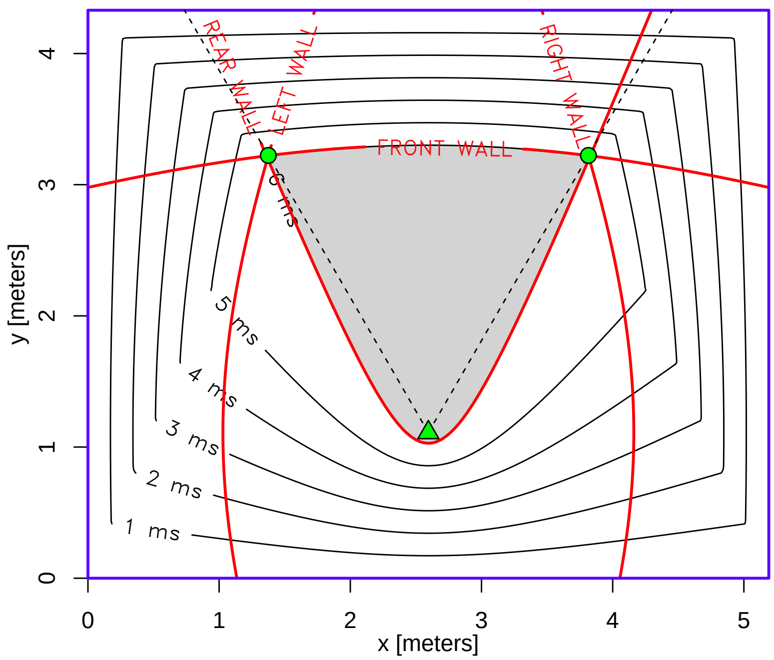


Figure 3: The smallest rectangular room with an 8 ft≈2.44 m equilateral arrangement of a listener (triangle) and loudspeakers (circles) that delays the arrival of the first lateral sound reflection by 6 ms. Black contours indicate, as a function of loudspeaker placement, the delay in arrival of the first reflection at the indicated listening position. Loudspeakers must be placed in the shaded region to achieve at least 6 ms delay. The 6 ms contours for reflections from individual walls are shown in red.

6 ms delay. Also shown, for each of the four walls, is the contour giving loudspeaker placements that delay a particular wall reflection by exactly 6 ms. This situation is highly constrained: moving either loudspeaker closer to a room boundary will cause one of the reflections to arrive with less than 6 ms delay. In the configuration shown all four room reflections arrive at the listener at the same time, which is perhaps undesirable. In a larger room it would be possible to stagger the reflection arrivals.

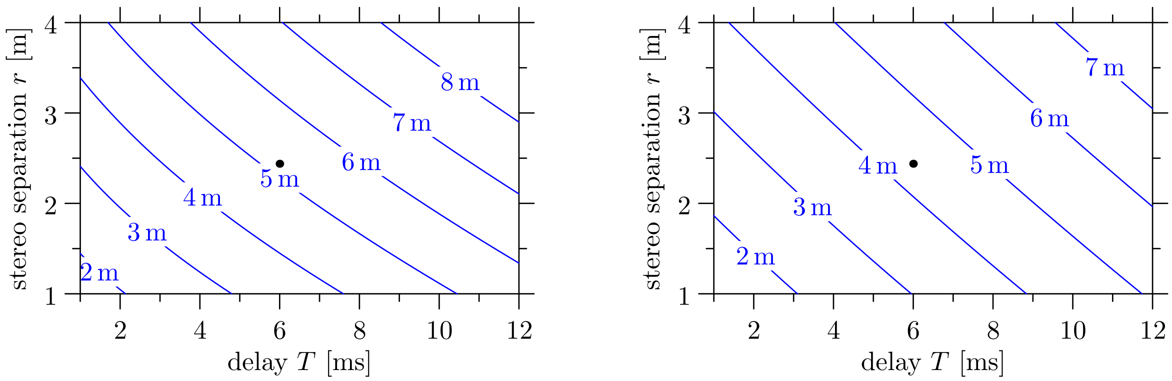
4 Equilateral Placement: General Case

Increasing the stereo separation r increases the minimum room dimensions needed to achieve a given first lateral reflection delay. Carrying out the preceding analysis in general, equation (2) gives the minimum room width

$$W = 2x = \tfrac{1}{2}r + \sqrt{\tfrac{1}{4}r^2 + 2rcT + c^2T^2} \qquad (u = \tfrac{1}{2}r), \tag{3}$$

and depth

$$D = 2x - u = \sqrt{\tfrac{3}{4}r^2 + 2rcT + c^2T^2} \qquad (u = \tfrac{\sqrt{3}}{2}r), \tag{4}$$



(a) Side Wall Reflections

(b) Front/Rear Wall Reflections

Figure 4: For loudspeakers and listener forming an equilateral triangle of side length r: (a) Contour plot of the minimum room width, W, for which it is possible to delay side wall reflections by an amount T (equation (3)). (b) Contour plot of the minimum room depth, D, for which it is possible to delay both front- and rear-wall reflections by T (equation (4)). The dots correspond to the particular case $r = 8\,\text{ft}$, $T = 6\,\text{ms}$ analyzed in Section 3.

for which it is possible to achieve a given reflection delay T from the front/rear and side walls, respectively, for a given stereo separation r in an equilateral arrangement. Figure 4 shows contour plots of these functions: each curve gives the relationship between stereo separation, r, and the maximum achievable first reflection delay, T, for a room of a given width or depth.

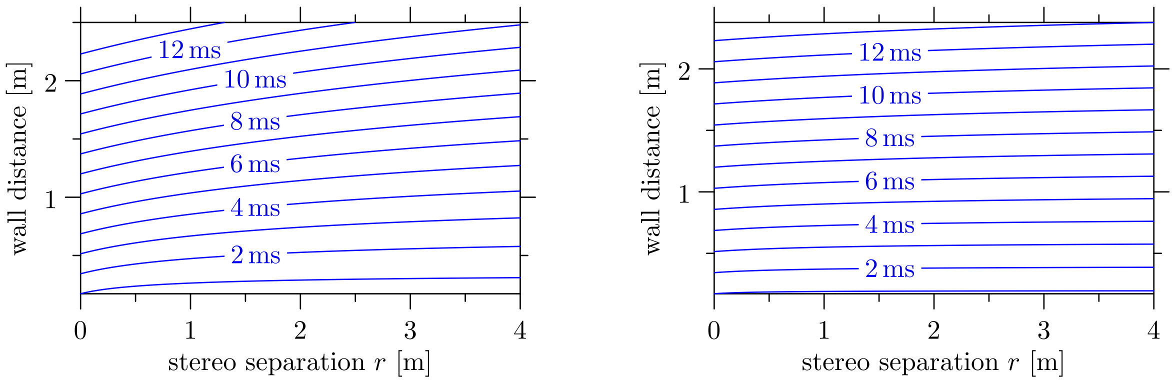
In Figure 4(a), for example, we see that in a 4 m-wide room the stereo separation must be reduced to about 1 m to achieve 6 ms delay for reflections from the side walls. For a room 4 m deep, Figure 4(b) shows that for a stereo separation of about 2 m the front- and rear-wall reflections can be delayed by up to 6 ms, provided the loudspeaker-listener triangle is properly placed with respect to front and rear walls.

Note that while the listening position is always symmetric with respect to the side walls, this is not the case for the front-to-rear position. The front- and rear-wall reflections are simultaneously maximally delayed when both reflections have the same delay. This occurs when the speaker-to-front-wall and listener-to-back-wall distances are equal. Consequently, when the stereo separation is comparable to the room dimensions (which is typically the case in small rooms) the listening position must move closer to the rear of the room to maximize both front- and rear-wall reflection delays simultaneously.

For any given room, equation (2) can be used to find the distance from speaker to front wall that will achieve a given delay for front wall reflections:

$$\text{front wall distance} = x - u = \left(\sqrt{\tfrac{3}{4}r^2 + 2rcT + c^2T^2} - \frac{\sqrt{3}}{2}r\right)/2 \qquad (u = \tfrac{\sqrt{3}}{2}r). \tag{5}$$

This expression also gives the distance from listener to rear wall to achieve a rear-wall reflection delay of T. Similarly, achieving a side-wall reflection delay of T requires that the distance from



(a) Side Wall Reflections

(b) Front/Rear Wall Reflections

Figure 5: For loudspeakers and listener forming an equilateral triangle of side length r: (a) Distance from speaker to side wall needed to achieve a given side wall reflection delay (equation (6)). (b) Distance from speaker to front wall (or listener to rear wall) to achieve a given reflection delay for these walls (equation (5)).

loudspeaker to side wall be

$$\text{side wall distance} = x - u = \left(\sqrt{\tfrac{1}{4}r^2 + 2rcT + c^2T^2} - \frac{1}{2}r\right)/2 \qquad (u = \tfrac{1}{2}r). \tag{6}$$

Figure 2 show graphs of relationships (5)–(6). The required wall distance depends only weakly on r, particularly for the front- and rear-wall reflections: speaker- and listener-to-wall distances of about 1.1 m $\approx$ 3.6 ft will achieve 6 ms delay for these reflections, essentially independent of the stereo separation.

5 Optimal Toe-In Angle for a Dipole

5.1 Minimizing Lateral Reflection Levels

A directional loudspeaker (e.g. dipole radiator) offers the possibility to orient the loudspeaker to selectively attenuate lateral room reflections, thus taking advantage of the level dependence of the precedence effect. This can be especially helpful in small rooms with too little space to obtain the desired 6 ms reflection delay.

Figure 6 shows two scenarios for the front- and nearest side-wall reflections with loudspeakers arranged in an 8 ft equilateral triangle (the loudspeaker is 1.1 m and and 1.4 m from the front and side walls, respectively), with the polar response of an ideal dipole superimposed. In Figure 6(a) the dipole axis is oriented directly toward the listener, as would typically be the case. The side wall reflection is severely attenuated, since it is radiated very near the dipole null (at about 88° off-axis, the attenuation is $-20\log_{10}(\cos 88°) \approx 29$ dB). The front wall reflection is also attenuated, but much less so (about 3 dB).

By rotating the dipole within this configuration one can vary the levels of the two first reflections, increasing one as the other decreases. It would seem, in the case of similar delays for the two

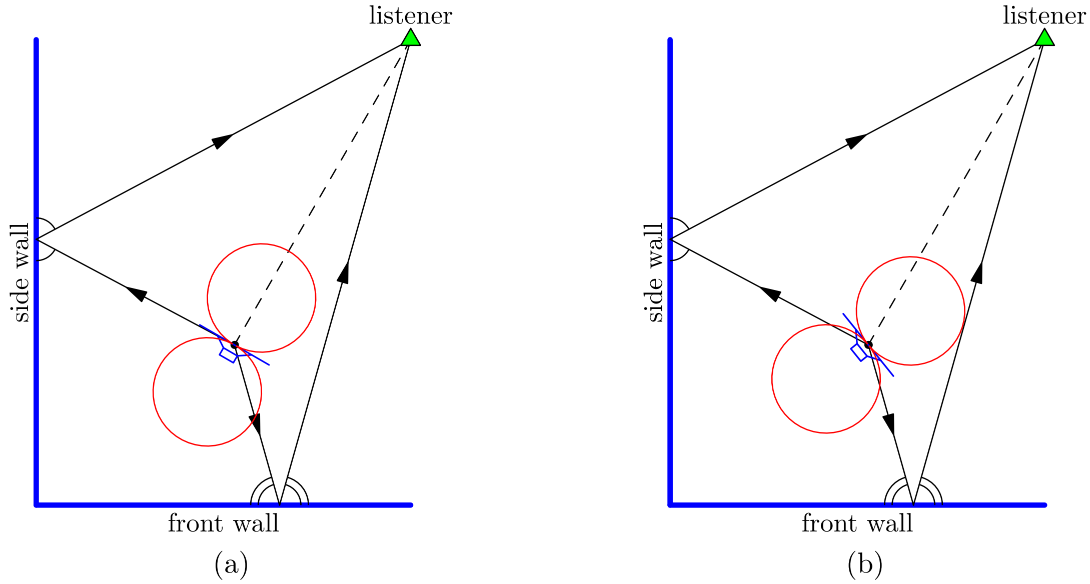


Figure 6: Reflection paths for first-order front- and side-wall reflections, with the polar response for an ideal dipole radiator superimposed (red). In (a) the dipole axis is oriented directly toward the listener, resulting in strong attenuation of the side wall reflection but much less attenuation of the front wall reflection. In (b) the dipole axis has been toed in to achieve equal radiated levels for the front- and side-wall reflections.

reflections, that the optimal toe-in angle should attenuate the reflections equally: any other angle will cause one of the reflections to increase in level. Neglecting the difference in attenuation (on the order of 1 dB) due to the difference between the reflection path lengths, this will occur when the dipole axis bisects the angle between the two rays that are reflected to the listener, as illustrated in Figure 6(b). Here the loudspeaker has been toed in by 21°, so that both reflections are radiated at 67° off-axis and therefore at equal levels of $-20\log_{10}(\cos 67°) \approx 8\,\text{dB}$ below the direct sound. In practice such an arrangement is easy to achieve: mirrors on the walls at the two reflection points will show symmetric views of the loudspeaker.

Here we derive a formula for the toe-in angle that places side- and front-wall reflections at equal angles relative to the dipole axis, thereby attenuating these reflections equally. Figure 7 illustrates the geometry. The red line segment indicates the dipole axis, which has been toed in by an angle ϕ relative to the listener. The listener is at (x_l, y_l) and the loudspeaker at (x_s, y_s) relative to the origin at corner. By extending rays from the loudspeaker through the reflection points R_1 and R_2 to the images of the listener in the walls, we obtain

$$\theta_1 = \tan^{-1}\frac{y_l - y_s}{x_l + x_s}, \qquad \theta_2 = \tan^{-1}\frac{x_l - x_s}{y_l + y_s}. \tag{7}$$

In order that the dipole axis bisects the angle between these rays, we require

$$\beta = \frac{1}{2}(\theta_1 + \theta_2 + 90°). \tag{8}$$

The dipole orientation α relative to the room is then given by

$$\alpha = \beta - \theta_1, \tag{9}$$

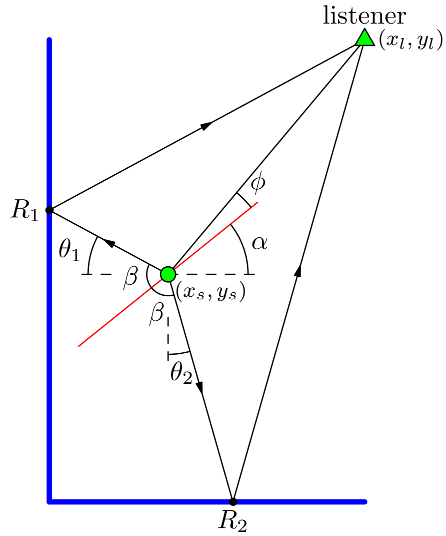


Figure 7: Geometry of front- and side-wall reflection paths resulting in both reflections being radiated at the same angle β from the dipole axis (red), which is toed in by an angle ϕ relative to the listener.

which after substituting from equation (7) and simplifying yields

$$\begin{aligned}\alpha &= 45^\circ + \frac{1}{2}(\theta_2 - \theta_1) \\ &= 45^\circ + \frac{1}{2}\tan^{-1}\frac{x_l - x_s}{y_l + y_s} - \frac{1}{2}\tan^{-1}\frac{y_l - y_s}{x_l + x_s} \\ &= 45^\circ + \frac{1}{2}\tan^{-1}\frac{y_s^2 - y_l^2 - x_s^2 + x_l^2}{2(x_s y_s + x_l y_l)}. \end{aligned} \tag{10}$$

The toe-in angle ϕ relative to the listener is then

$$\begin{aligned}\phi &= \tan^{-1}\left(\frac{y_l - y_s}{x_l - x_s}\right) - \alpha \\ &= \tan^{-1}\frac{y_l - y_s}{x_l - x_s} - \frac{1}{2}\tan^{-1}\frac{y_s^2 - y_l^2 - x_s^2 + x_l^2}{2(x_s y_s + x_l y_l)} - 45^\circ. \end{aligned} \tag{11}$$

Equation (11) gives the desired toe-in angle ϕ as a rather complicated function of the loudspeaker and listener positions. For illustration, consider a fixed listener at $x_l = 2.6\,\mathrm{m}$, $y_l = 3.2\,\mathrm{m}$. Figure 8(a) shows the corresponding toe-in angle as a function of loudspeaker placement, calculated by equation (11). Within the range of typical placements the optimal toe-in angle varies between about 10° and 30°. It appears to be a function mostly of the orientation of listener and loudspeaker, and largely independent of their separation.

With the dipole axis toed in as described above, the ratio of reflected to direct sound level can be determined as a function of loudspeaker placement. The reflected and direct sounds are radiated at off-axis angles β and ϕ, respectively (Figure 7), hence for an ideal dipole the ratio of reflected

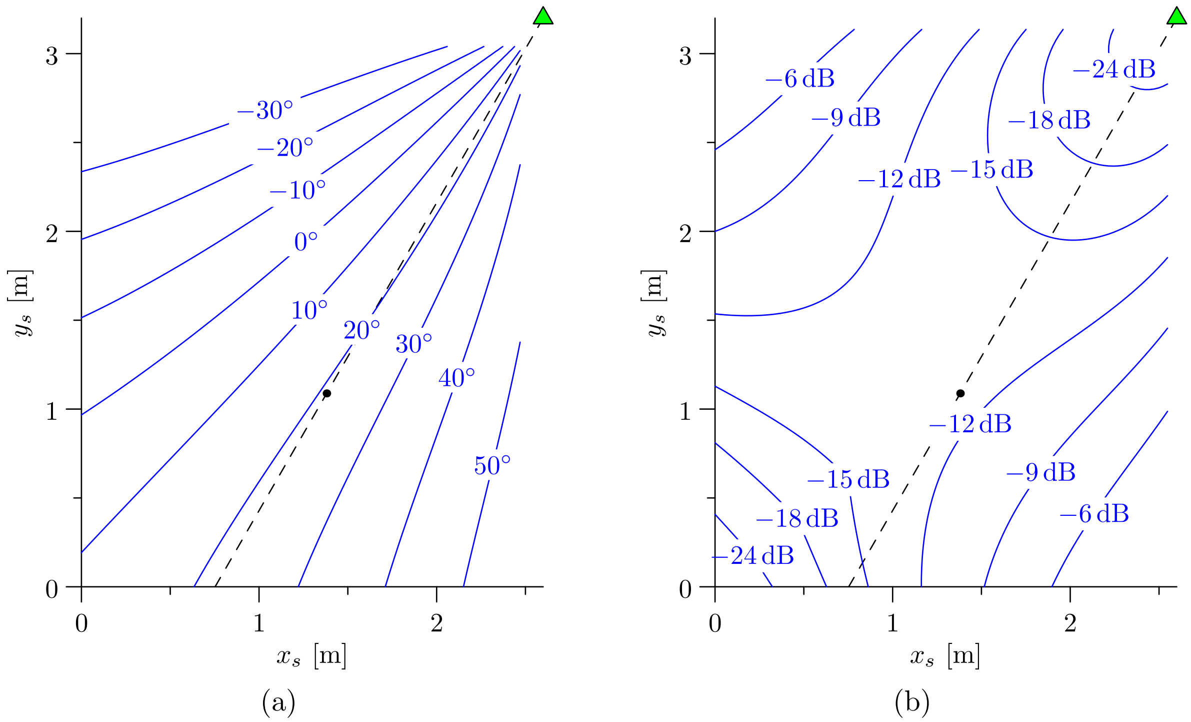


Figure 8: For a fixed listener position (green triangle) at $x_l = 2.6\,\text{m}$, $y_l = 3.2\,\text{m}$: (a) Toe-in angle ϕ (see Figure 7) resulting in equal radiated levels for front- and side-wall first reflections, as a function of loudspeaker placement. (b) Ratio, at the listening position, of front- or side-wall reflected sound level (whichever is higher) to direct sound, for an ideal dipole toed in according to (a). For reference the dashed line indicates loudspeaker placements for an equilateral arrangement of a stereo pair along the front wall; the dot indicates placement for 8 ft stereo separation.

to direct sound level will be

$$\frac{\text{reflected radiation}}{\text{direct radiation}} = \frac{\cos\beta}{\cos\phi} \tag{12}$$

at the source, with β and ϕ given by equations (8) and (11). Because sound levels decrease as the reciprocal of path length, there will be a further attenuation factor of

$$\left(\frac{\text{reflection attenuation}}{\text{direct attenuation}}\right)_{\text{front}} = \sqrt{\frac{(x_l - x_s)^2 + (y_l - y_s)^2}{(x_l - x_s)^2 + (y_l + y_s)^2}} \tag{13}$$

for the front wall reflection and

$$\left(\frac{\text{reflection attenuation}}{\text{direct attenuation}}\right)_{\text{side}} = \sqrt{\frac{(x_l - x_s)^2 + (y_l - y_s)^2}{(x_l + x_s)^2 + (y_l - y_s)^2}} \tag{14}$$

for the side wall reflection, relative to the direct sound.

Figure 8(b) shows, as a function of loudspeaker placement, the net ratio of reflected to direct sound as calculated from equations (12)–(14), for the same room geometry and dipole toe-in as in Figure 8(a). Interestingly, typical loudspeaker placements fall near the saddle point, so that the reflected-to-direct ratio is largely insensitive to placement. For placement nearer to the listener the ratio is dominated by the decrease in direct sound level at 6 dB per doubling of distance, as expected. More surprising is that the direct- to reflected-sound ratio increases rapidly as the loudspeaker is placed nearer to the corner of the room. This is because corner placement puts both reflection paths in the dipole null (achieved in this instance with a toe-in angle of $\phi \approx 12°$).

In smaller or more constrained rooms, where the loudspeakers must be placed closer to either the front or side walls, one can selectively suppress the first reflection from the closest wall by adjusting the toe-in angle. In this case it is unclear what criterion should be used to determine the optimal toe-in angle. A determination would require accurate data on how the precedence effect depends on both level and delay, for delays in the range of 1-10 ms.

5.2 Minimizing Inter-Channel Level Difference for Off-Center Listeners

When a listener is placed off-center with respect to a stereo pair of loudspeakers, the sound level due to the closer loudspeaker will be higher. If the resulting level difference between speakers is great enough, the phantom stereo image will be perceived to collapse into the nearest loudspeaker. This phenomenon limits the size of the "sweet spot" in which the stereo illusion is stable. By toeing-in a dipole stereo pair one can reduce the inter-channel level difference for off-center listeners, thus widening the sweet spot: as the listener moves laterally away from one loudspeaker, the decrease in sound level due to increased distance is compensated by the increase in level closer to the dipole axis. A judicious choice of toe-in angle will balance these effects exactly, resulting in a constant left/right balance as the listener moves across the center position.

Figure 9 illustrates the geometry when a listener (green triangle) is offset a lateral distance x from the center listening position with respect to a pair of loudspeakers (green circles). The dipole axis (red line segment) is toed in by an angle ϕ relative to the center position (black dot). We wish to determine the toe-in angle ϕ that minimizes the variation of inter-channel level difference as a function of x.

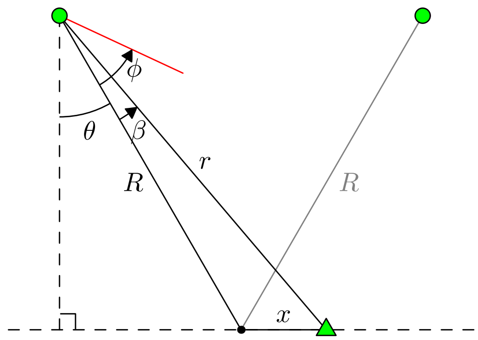


Figure 9: Geometry of the direct sound path when a listener (green triangle) is displaced laterally a distance x from center with respect to a stereo pair of loudspeakers (green circles), each of which is toed in by an angle ϕ relative to center (black dot). The red line segment indicates the loudspeaker axis.

For a single ideal dipole at the left loudspeaker, the sound level at the listener relative to the level at center is

$$\begin{aligned} A_l &= \frac{R}{r}\cos(\phi - \beta) \\ &= \frac{R}{r}\big(\cos\phi\cos\beta + \sin\phi\sin\beta\big). \end{aligned} \tag{15}$$

To express equation (15) as a function of x only, we use the cosine and sine laws respectively to write

$$x^2 = R^2 + r^2 - 2Rr\cos\beta \implies \cos\beta = \frac{R^2 + r^2 - x^2}{2Rr}$$

and

$$\frac{\sin\beta}{x} = \frac{\sin(\frac{\pi}{2} + \theta)}{r} = \frac{\cos\theta}{r} \implies \sin\beta = \frac{x}{r}\cos\theta.$$

Substituting these expressions into equation (15) yields

$$A_l = \frac{R}{r^2}\left(\frac{R^2 + r^2 - x^2}{2R}\cos\phi + x\cos\theta\sin\phi\right).$$

Finally, since

$$\begin{aligned} r^2 &= (x + R\sin\theta)^2 + (R\cos\theta)^2 \\ &= x^2 + R^2 + 2xR\sin\theta, \end{aligned}$$

we obtain (after some simplification)

$$A_l = \frac{(1 + \frac{x}{R}\sin\theta)\cos\phi + \frac{x}{R}\cos\theta\sin\phi}{1 + (\frac{x}{R})^2 + 2\frac{x}{R}\sin\theta}. \tag{16}$$

By symmetry we can exchange x for $-x$ to obtain the level from the right loudspeaker:

$$A_r = \frac{(1 - \frac{x}{R}\sin\theta)\cos\phi - \frac{x}{R}\cos\theta\sin\phi}{1 + (\frac{x}{R})^2 - 2\frac{x}{R}\sin\theta}. \tag{17}$$

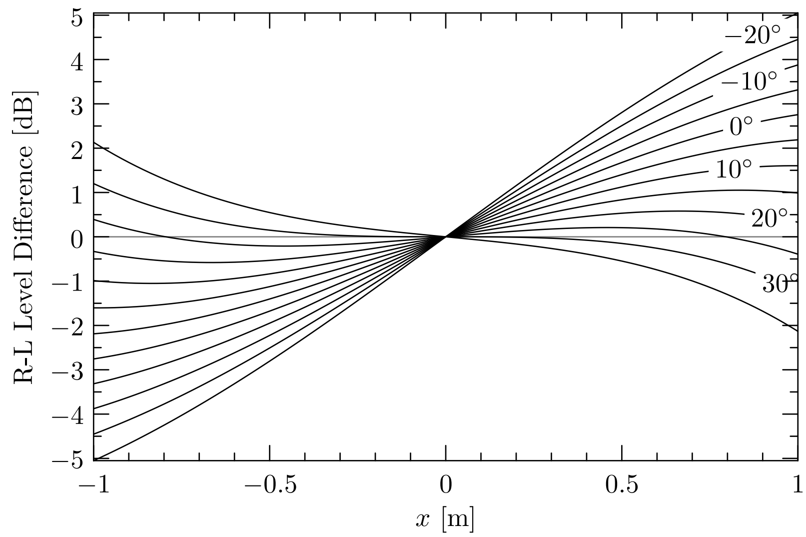


Figure 10: Inter-channel level difference as a function of lateral displacement of the listener from center, for various values of the toe-in angle ϕ of a stereo pair of ideal dipole radiators in an equilateral arrangement with stereo separation $R = 8\,\text{ft} \approx 2.4\,\text{m}$. (c.f. Figure 9 and equations (16)–(17).)

For a typical 8 ft equilateral arrangement ($R = 8\,\text{ft} \approx 2.4\,\text{m}$, $\theta = 30°$), Figure 10 shows graphs of the inter-channel level difference A_l/A_r as a function of listener offset x, for several values of the toe-in angle ϕ. As the toe-in angle increases, the inter-channel level difference decreases at all off-center listening positions. For a range of toe-in angles between about 15° and 30° the inter-channel difference is less than 1 dB across a 2 m-wide listening region.

In Figure 10 the maximally flat response at center is achieved when $\phi = 30°$. Indeed, it is straightforward to show from equations (16)–(17) that in general the conditions for maximally flat response at center, *viz*

$$\left.\frac{dA_l}{dx}\right|_{x=0} = 0 = \left.\frac{dA_r}{dx}\right|_{x=0},$$

are met if and only if $\phi = \theta$.

An interesting feature of Figure 10 is that for toe-in angles near 25° there are three distinct "sweet spots" at which left and right levels are exactly matched: the center position and two positions located symmetrically to either side of center. Using equations (16)–(17) and setting $A_l = A_r$ yields (after some simplification)

$$x\big[R^2(\tan\theta - \tan\phi) - x^2(\tan\theta + \tan\phi)\big] = 0. \tag{18}$$

Solving for x yields the locations of the three sweet spots,

$$x_{\text{equal}} = 0 \quad \text{or} \quad x_{\text{equal}} = \pm R\sqrt{\frac{\tan\theta - \tan\phi}{\tan\theta + \tan\phi}}. \tag{19}$$

The secondary sweet spots exist only when $\phi < \theta$.

Alternatively, solving equation (18) for ϕ gives

$$\phi = \tan^{-1}\left(\frac{R^2 - x_{\mathrm{equal}}^2}{R^2 + x_{\mathrm{equal}}^2}\tan\theta\right) \tag{20}$$

for the toe-in angle ϕ that places the secondary sweet spots at desired off-center locations. For example with $R = 8\,\mathrm{ft} \approx 2.4\,\mathrm{m}$, if we wish to place the secondary sweet spots at $x_{\mathrm{equal}} = \pm 0.5\,\mathrm{m}$ (thus providing sweet spots for a pair of listeners seated 1 m apart) equation (20) gives the corresponding toe-in angle $\phi \approx 28°$. Inter-channel timing differences will be compromised at the secondary sweet spots, but at least level differences will not cause the stereo image to collapse to one side.

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