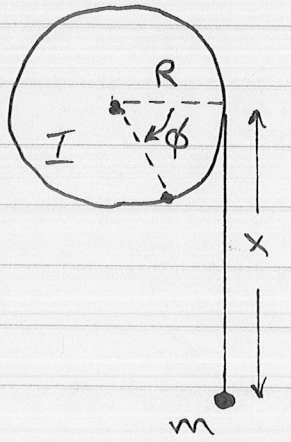
7.52




$$x = R\phi \rightarrow \underbrace{x - R\phi = 0}_{f(x,\phi)}$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\phi}^2$$

$$U = -mgx$$

$$\rightarrow L = T - U = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\phi}^2 + mgx$$

Equations of motion:

$$\frac{\partial L}{\partial x} + \lambda\frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} \rightarrow mg + \lambda = m\ddot{x} \quad (1)$$

↳ tension $= -\lambda$

$$\frac{\partial L}{\partial \phi} + \lambda\frac{\partial f}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} \rightarrow -\lambda R = I\ddot{\phi} \quad (2)$$

↳ torque on wheel

$$\ddot{x} = R\ddot{\phi} \quad (3)$$

$$(3) \rightarrow \left(g + \frac{\lambda}{m}\right) = R\left(-\frac{\lambda R}{I}\right)$$

$$\rightarrow \left(\frac{1}{m} + \frac{R^2}{I}\right)\lambda = -g \rightarrow \lambda = -\left(\frac{1}{m} + \frac{R^2}{I}\right)^{-1} g$$

$$= -\frac{mI}{I + mR^2}\cdot g$$

(1) → $\boxed{\ddot{x} = g - \frac{I}{I+mR^2} g = \frac{mR^2}{I+mR^2} \cdot g}$

(2) → $\boxed{\ddot{\phi} = \frac{mR}{I+mR^2} \cdot g}$