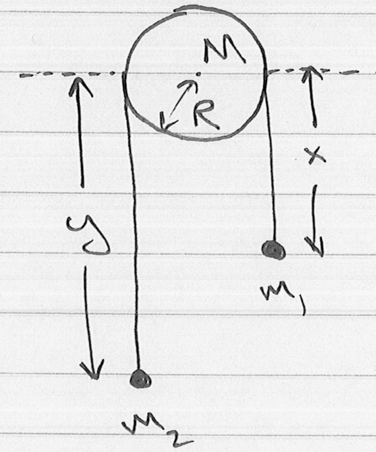
13.3 Atwood's machine with uniform pulley: $I = \frac{1}{2}MR^2$



$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{y}^2 + \frac{1}{2}I\omega^2$$

but $\dot{y} = -\dot{x}$ and $\omega = \frac{\dot{x}}{R}$ so:

$$T = \frac{1}{2}(m_1+m_2)\dot{x}^2 + \frac{1}{4}M\dot{x}^2$$

$$U = -m_1 g x - m_2 g y = -m_1 g x - m_2 g(l-x)$$

$$\rightarrow L = T - U$$
$$= \frac{1}{2}(m_1+m_2)\dot{x}^2 + \frac{1}{4}M\dot{x}^2 + m_1 g x + m_2 g(l-x)$$

generalized momentum:

$$p = \frac{\partial L}{\partial \dot{x}} = (m_1+m_2)\dot{x} + \frac{1}{2}M\dot{x} = \left(m_1+m_2+\frac{1}{2}M\right)\dot{x}$$

Hamiltonian:

$$H = p\dot{q} - L = p\dot{x} - L$$

$$= p\dot{x} - \frac{1}{2}\underbrace{\left(m_1+m_2+\frac{M}{2}\right)\dot{x}}_{p}{}^{2} - m_1 g x - m_2 g(l-x)$$

$$= \frac{1}{2}p\dot{x} - m_1 g x - m_2 g(l-x)$$

$$= \frac{\frac{1}{2}p^2}{m_1+m_2+\frac{M}{2}} - m_1 g x - m_2 g(l-x)$$

Equations of motion:

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m_1 + m_2 + \frac{M}{2}} \; ; \; \dot{p} = -\frac{\partial H}{\partial x} = m_1 g - m_2 g$$

$$\rightarrow \ddot{x} = \frac{\dot{p}}{m_1 + m_2 + \frac{M}{2}} = \frac{(m_1 - m_2) g}{m_1 + m_2 + M/2}$$