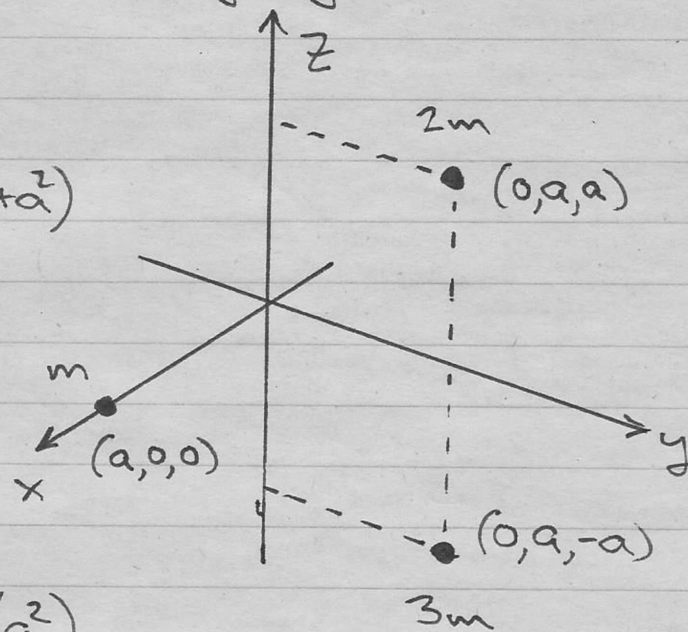
10.35 3 point masses connected rigidly as shown:



a) $I_{xx} = \sum_i m_i (y_i^2 + z_i^2)$

$= (m)(0) + (2m)(a^2+a^2) + 3m(a^2+a^2)$

$= 10ma^2$

$I_{yy} = \sum_i m_i (x_i^2 + z_i^2)$

$= (m)(a^2) + (2m)(a^2) + (3m)(a^2)$

$= 6ma^2$

$I_{zz} = \sum_i m_i (x_i^2 + y_i^2) = (m)(a^2) + (2m)(a^2) + (3m)(a^2)$

$= 6ma^2$

$I_{xy} = -\sum_i m_i x_i y_i = -\left[(m)(0) + (2m)(0) + (3m)(0)\right]$

$= 0$

$I_{xz} = -\sum_i m_i x_i z_i = -\left[(m)(0) + (2m)(0) + (3m)(0)\right]$

$= 0$

$I_{yz} = -\sum_i m_i y_i z_i = -\left[(m)(0) + (2m)(a^2) + (3m)(-a^2)\right]$

$= ma^2$

$$\therefore \quad I = ma^2 \begin{bmatrix} 10 & 0 & 0 \\ 0 & 6 & 1 \\ 0 & 1 & 6 \end{bmatrix}$$

b) Principal moments (eigenvalues of I):

$$0 = \det(I - \lambda \mathbb{1}) = \begin{vmatrix} 10-\lambda & 0 & 0 \\ 0 & 6-\lambda & 1 \\ 0 & 1 & 6-\lambda \end{vmatrix} = (10-\lambda)\left[(6-\lambda)^2 - 1\right]$$

$\Rightarrow \lambda = 10$ or $\lambda = 6 \pm 1$ ... now find eigenvectors $\vec{v}$:

<u>case $\lambda = 5$:</u>

$$I - \lambda \mathbb{1} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(I - \lambda \mathbb{1})\vec{v} = \vec{0} \Rightarrow \vec{v} = v_z \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad (v_z \in \mathbb{R})$$

<u>case $\lambda = 7$:</u>

$$I - \lambda \mathbb{1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \vec{v} = v_z \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad (v_z \in \mathbb{R})$$

<u>case $\lambda = 10$:</u>

$$I - \lambda \mathbb{1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -4 & 1 \\ 0 & 1 & -4 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \vec{v} = v_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (v_x \in \mathbb{R})$$

So the (orthogonal) principal axes and corresponding moments are:

$\lambda_1 = 10 \quad \vec{v}_1 = (1, 0, 0)$

$\lambda_2 = 5 \quad \vec{v}_2 = (0, 1, -1)$

$\lambda_3 = 7 \quad \vec{v}_3 = (0, 1, 1)$