Instructions:

1. You have 24 hours to complete the exam.
2. Write your solutions on your own paper, scan or photograph them (scan to a single PDF document is strongly preferred) and email them to `rtaylor@tru.ca` by 15 April 10:00AM PST. Late submissions will not be accepted.
3. You may use any computational tools at your disposal (e.g. calculator, Wolfram Alpha, computer algebra software) to simplify calculations, but be careful to indicate where and why you have done so. In any case, you must clearly explain your reasoning to receive full credit.
4. Organization and neatness count.
5. Include the following signed and dated Declaration of Academic Integrity on the first page of your submission (feel free to just print and sign this page):

   By submitting this work for assessment I hereby declare that it is the result of my own effort and that I did not copy (in whole or in part) the work of any other individual.

   I also declare that subsequent to receiving the assigned work I did not discuss the questions or possible answers with any other person, either face to face or electronically.

   By submitting this declaration I agree to any reasonable level of scrutiny deemed necessary to determine whether I have violated TRU Policy on Student Academic Integrity ED 5-0.

Name: ______________________ **Signature:** ______________________

Student #: ______________________ **Date:** ______________________

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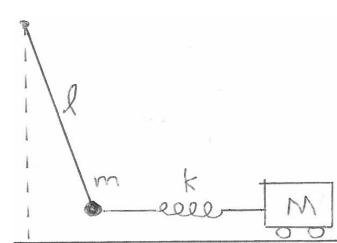
Problem 1: A pendulum (point mass m suspended from a fixed support by a light, rigid rod of length l) is connected to a cart of mass M by a spring with stiffness k, as shown in the diagram. The spring is long, so that when the pendulum makes small oscillations the spring remains horizontal, to a good approximation. The cart rolls without friction on a flat, horizontal surface.

(a) Write the Lagrangian function for this system. Be sure to clearly define your chosen coordinates.

(b) Find and simplify the equations of motion.

(c) For the case $m = M = g = l = k = 1$, find the frequencies of small oscillations and describe the corresponding normal modes.



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Problem 2: A point mass m, initially at rest, falls (without friction) to the ground from a height h. It lands some distance D (the "Coriolis deflection") from where it would have landed in the absence of the Coriolis force.

(a) Calculate D and its direction. Your answer might depend on the co-latitude θ where the experiment is performed. How would your answer change if the experiment were performed at the same latitude in the Southern hemisphere?

(b) Calculate the ratio h/D at $\theta = 40°$. How large must h be in order to obtain a Coriolis deflection of $D = 1\,\text{mm}$?

(c) Suppose instead that the same mass is thrown vertically upward from the ground, with initial speed v_0 chosen so that it rises to the same maximum height h before falling back to the ground. Show that it lands a distance $4D$ from where it left the ground, but in the *opposite direction* from part (a).

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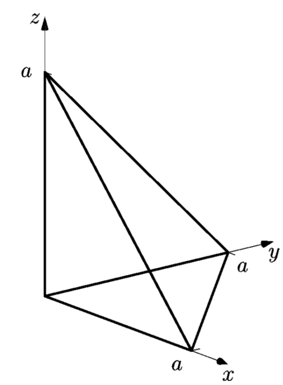
Problem 3: Consider a rigid solid object with uniform mass density in the shape of a tetrahedron with vertices at $O = (0, 0, 0)$, $(a, 0, 0)$, $(0, a, 0)$ and $(0, 0, a)$, as shown in the diagram. Consider rotation of this object about an axis through O.

(a) Calculate the moment of inertia tensor.

(b) Calculate the angular momentum vector $\mathbf{L}$ for rotation about the z-axis with angular speed ω. In the absence of an external torque, can the object spin about the z-axis without wobbling? Explain.

(c) Calculate the principal axes and corresponding moments of inertia.



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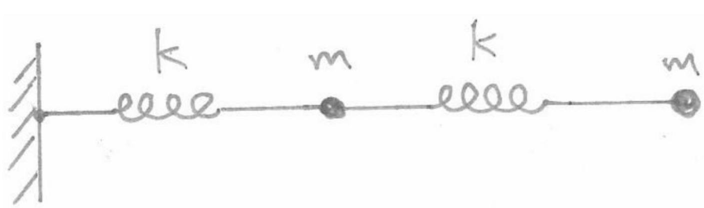
Problem 4: Two identical point masses are constrained to move horizontally in one dimension, without friction. One mass is connected by a spring (with rest length l_0 and stiffness k) to a fixed support. The second mass is connected to the first by an identical spring, as shown in the diagram.



(a) Write the Hamiltonian function for this system. Use coordinates x and y such that the lengths of the springs are $l_0 + x$ and $l_0 + y$, respectively.

(b) Find all four of Hamilton's equations of motion.

(c) Hamilton's equations are linear. With $\mathbf{v} = (x, p_x, y, p_y)$ they can be written in matrix form as $\mathbf{v}' = A\mathbf{v}$. Find the matrix A.

(d) Find all possible values of the frequency ω of oscillatory solutions, by substituting the solution $\mathbf{v}(t) = \mathbf{z}e^{i\omega t}$ into the differential equation in part (c) and solving for the values of ω that yield a solution with non-zero constant vector $\mathbf{z}$