**Name:**____________________________

**Signature:**____________________________

**Instructions:**

1. Prepare your solutions on your own paper, stapled with this exam paper at the front.
2. It is permissible to discuss the problems with other students or faculty members. However, your written solutions must be exclusively your own work. The TRU Policy on Academic Integrity (`http://www.tru.ca/__shared/assets/ed05-05657.pdf`) will be strictly enforced.
3. You must clearly show your work and explain your reasoning to receive full credit.
4. Organization and neatness count.
5. Submit your solutions to Joan Silbernagel or Marcy Desrosiers in the Mathematics & Statistics Department reception area **before 12:00 noon 27 April 2012**.
6. Late submissions will not be accepted.

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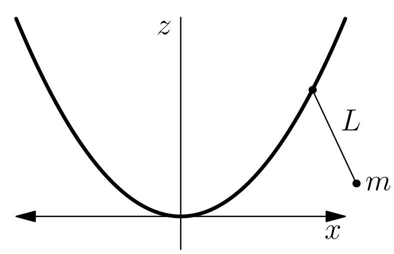
**Problem 1:** The point of suspension of a simple pendulum (a point mass $m$ suspended by a massless rigid rod of length $L$) is constrained to move without friction along the parabola $z = ax^2$ $(a > 0)$ in the vertical $(x, z)$-plane.

(a) Derive the equations of motion for the system.

(b) Analyze this system further: what else can we learn about its motion?

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**Problem 2:** A uniform bar of mass $M$ and length $2L$ is suspended from one end by a spring of force constant $k$. The bar can swing freely only in one vertical plane, and the spring is constrained to move only in the vertical direction.

(a) Set up the equations of motion in the Lagrangian formulation.

(b) Find the Hamiltonian and obtain Hamilton's equations of motion for the system.

(c) What are the normal frequencies for small oscillations about the equilibrium where the bar hangs vertically downward? Find and describe the normal modes.

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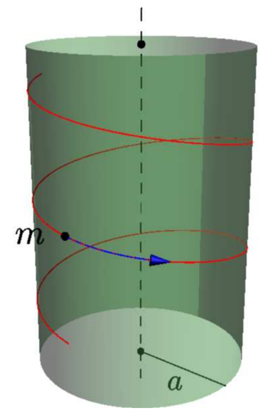
**Problem 3:** A uniform cylinder of radius $a$ and density $\rho$ is mounted so as to rotate freely about its axis, which is vertical. On the outside of the cylinder is a rigidly fixed uniform spiral or helical track along which a point mass $m$ can slide without friction. Suppose the point mass starts at rest at the top of the cylinder and slides down under the influence of gravity.




(a) Use the method of Lagrangian constraints to find the equations of motion for the system.

(b) Solve for and discuss the physical meaning of the Lagrange multipliers.

(c) Note any conserved quantities.

(d) Solve for the motion of the system.

(e) Solve for the motion again, this time by deriving and solving Hamilton's equations of motion in suitable generalized coordinates.

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**Problem 4:** A point particle is thrown up vertically from the ground with initial speed $v_0$, reaches a maximum height and falls back to the ground. Due to the Coriolis force, the particle lands some distance (the "Coriolis deflection") from where it started. Show that the Coriolis deflection is four times greater in magnitude, but *opposite in direction* to the Coriolis direction when it is dropped from rest from the same maximum height.

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**Problem 5:** Consider a thin, flat rigid body in the shape of a 45° right triangle with uniform mass density.

(a) Find the moment of inertia tensor for rotation about the center of mass.

(b) Find the principal moments of inertia.

(c) What are the principal axes?