MATH 3160: Quiz #4 – SOLUTIONS

/5 **Problem 1:** Find $f(t) = \mathcal{L}^{-1}\left\{\dfrac{2s+5}{s^2+6s+34}\right\}$.

Complete the square:

$$F(s) = \frac{2s+5}{s^2+6s+34} = \frac{2s+5}{(s+3)^2+25} = \frac{2(s+3)-1}{(s+3)^2+25}$$

Then

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\left\{\frac{2(s+3)-1}{(s+3)^2+25}\right\} \\ &= e^{-3t}\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2+5}\right\} \\ &= e^{-3t}\mathcal{L}^{-1}\left\{2\cdot\frac{s}{s^2+5^2} - \frac{1}{5}\cdot\frac{5}{s^2+5^2}\right\} \\ &= \boxed{e^{-3t}\left[2\cos(5t) - \frac{1}{5}\sin(5t)\right]} \end{aligned}$$

/5 **Problem 2:** Find the solution $y(t)$ of the initial value problem $\begin{cases} y'' + 2y' = \delta(t-3) \\ y(0) = 0,\ y'(0) = 1. \end{cases}$

The Laplace transform of this IVP gives

$$(s^2Y - 1) + 2(sY) = e^{-3s} \implies (s^2+2s)Y = 1 + e^{-3s} \implies Y(s) = \frac{1}{s(s+2)} + \frac{e^{-3s}}{s(s+2)}$$

Partial fractions:

$$\frac{1}{s(s+2)} = \frac{1/2}{s} - \frac{1/2}{s+2}$$

$$\begin{aligned} \implies y(t) &= \mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\cdot\frac{1}{s+2} + \frac{1}{2}\cdot\frac{e^{-3s}}{s} - \frac{1}{2}\cdot\frac{e^{-3s}}{s+2}\right\} \\ &= \frac{1}{2} - \frac{1}{2}e^{-2t} + \frac{1}{2}u(t-3) - \frac{1}{2}u(t-3)e^{-2(t-3)} \\ &= \boxed{\frac{1}{2}\left(1 - e^{-2t} + u(t-3)\left[1 - e^{-2(t-3)}\right]\right)} \end{aligned}$$