MATH 2670: Quiz #5 – SOLUTIONS

$/5$ **Problem 1:** Find the solutions $y(x)$ of the following initial-value problem: $\begin{cases} y'' - 6y' + 9y = 0 \\ y(0) = 5,\ y'(0) = 10. \end{cases}$

First find the general solution by assuming $y = e^{rx}$:

$$r^2 - 6r + 9 = (r-3)^2 = 0 \implies r = 3 \text{ (repeated root)}$$

$$\implies y(x) = c_1 e^{3x} + c_2 x e^{3x} \quad (c_1, c_2 \in \mathbb{R})$$

Now impose the initial conditions:

$$\begin{aligned} &y(0) = 5 = c_1 + 0 \implies c_1 = 5 \\ &y'(0) = 10 = 3c_1 + c_2 = 3(5) + c_2 \implies c_2 = -5. \end{aligned}$$

$$\implies \boxed{y(x) = 5e^{3x} - 5xe^{3x}}$$

$/5$ **Problem 2:** Find the general solution $y(x)$ of the differential equation $y'' + 4y = 3\sin x$.

First solve the corresponding homogeneous problem $y'' + 4y = 0$. Assuming $y = e^{rx}$ gives

$$r^2 + 4 = 0 \implies r = \pm 2i \implies y_h(x) = c_1 \cos(2x) + c_2 \sin(2x).$$

To find a particular solution y_p, use undetermined coefficients and assume a solution of the form

$$\begin{aligned} y &= A\cos x + B\sin x \\ y' &= -A\sin x + B\cos x \\ y'' &= -A\cos x - B\sin x. \end{aligned}$$

Substituting these into the given DE gives

$$(-A\cos x - B\sin x) + 4(A\cos x + B\sin x) = 3\sin x.$$

Matching coefficients on the left- and right-hand sides gives:

$$\begin{cases} 3A = 0 \implies A = 0 \\ 3B = 3 \implies B = 1 \end{cases} \implies y_p = \sin x.$$

Now we can form the general solution:

$$y(x) = y_p(x) + y_h(x) = \boxed{\sin x + c_1\cos(2x) + c_2\sin(2x) \quad (c_1, c_2 \in \mathbb{R})}$$