MATH 2670: Quiz #1 – SOLUTIONS

/5 **Problem 1:** Evaluate the line integral $\int_C z\ln(x+y)\,ds$ where C has parametric equations $x = 1+3t$, $y = 2+t^2$, $z = t^4$ $(-1 \le t \le 1)$.

$$\mathbf{r}(t) = (1+3t, 2+t^2, t^4) \implies ds = |\mathbf{r}'(t)|\,dt = |(3, 2t, 4t^3)|\,dt = \sqrt{9+4t^2+16t^6}\,dt$$

$$\begin{aligned}\implies \int_C z\ln(x+y)\,ds &= \int_{-1}^{1} (t^4)\ln\big((1+3t)+(2+t^2)\big)\sqrt{9+4t^2+16t^6}\,dt \\ &= \int_{-1}^{1} t^4 \ln(3+3t+t^2)\sqrt{9+4t^2+16t^6}\,dt \approx \boxed{1.73}\end{aligned}$$

(this integral can't be evaluated analytically…I did it numerically on a computer)

/5 **Problem 2:** The vector field $\mathbf{F}(x,y) = x^2y^3\hat{\mathbf{i}} + x^3y^2\hat{\mathbf{j}}$ is conservative. Find a potential function $f(x,y)$ such that $\mathbf{F} = \nabla f$, and use it to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is any smooth curve from $(0,1)$ to $(1,0)$.

$$\frac{\partial f}{\partial x} = x^2y^3 \implies f(x,y) = \tfrac{1}{3}x^3y^3 + C(y)$$

$$\frac{\partial f}{\partial y} = x^3y^2 = x^3y^2 + C'(y) \implies C(y) = C\ (= 0, \text{ say})$$

$$\implies f(x,y) = \tfrac{1}{3}x^3y^3$$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = f(1,0) - f(0,1) = 0 - 0 = \boxed{0}$$