THOMPSON RIVERS UNIVERSITY

Department of Mathematics & Statistics

MATH 2670
Calculus 4 for Engineering

Section 01

Instructor: Richard Taylor

FINAL EXAM
(take-home)

20 April 2020 09:00–13:00

Instructions:

1. You have 4 hours to complete and submit the exam.
2. Write your solutions either on your own paper or on a printed copy of this exam. Scan or photograph your solutions (scan to a single PDF document is strongly preferred) and email them to `rtaylor@tru.ca` by 20 April 1:00PM PST. Late submissions will not be accepted.
3. Organization and neatness count.
4. Include the following signed and dated Declaration of Academic Integrity on the first page of your submission (feel free to just print and sign this page and include it with your submission):

   By submitting this work for assessment I hereby declare that it is the result of my own effort and that I did not copy (in whole or in part) the work of any other individual.

   I also declare that subsequent to receiving the assigned work I did not discuss the questions or possible answers with any other person, either face to face or electronically.

   By submitting this declaration I agree to any reasonable level of scrutiny deemed necessary to determine whether I have violated TRU Policy on Student Academic Integrity ED 5-0.

Name: ____________________ **Signature:** ____________________

Student #: ____________________ **Date:** ____________________

/6 **Problem 1:** Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y) = xy\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$$

and C is the graph of $x = y^2$ from $(0, 0)$ to $(1, 1)$.

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Problem 2: Consider the vector field $\mathbf{F}(x, y) = 2xy^3\hat{\mathbf{i}} + 3y^2x^2\hat{\mathbf{j}}$.

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(a) Show that $\mathbf{F}$ is conservative and find $f(x, y)$ such that $\mathbf{F} = \nabla f$.

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(b) Evaluate the path integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the unit circle $x^2 + y^2 = 1$, oriented clockwise.

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(c) Evaluate the path integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the part of the circle $x^2 + y^2 = 1$ that lies in the first quadrant, oriented clockwise.

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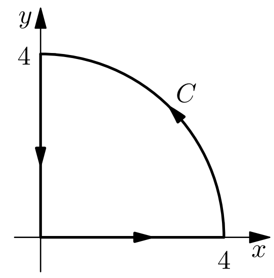
Problem 3: Use Green's theorem to evaluate the path integral

$$\oint_C x^2y\,dx - y^2x\,dy$$

where C is the boundary of the region in the first quadrant enclosed by the x- and y-axes and the circle $x^2 + y^2 = 16$.



/6 **Problem 4:** Use the Divergence Theorem to evaluate the surface integral $\oint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}$ is the vector field $\mathbf{F}(x,y,z) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ and S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.

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Problem 5: Find the general solution $y(x)$ of the following differential equations:

(a) $y' + 2xy^2 = 0$

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(b) $xy' - y = x^3 - x$

/3

(c) $y'' - y' = \sin(2x)$

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Problem 6: A tank initially contains 100 gal of fresh water. Then water containing 0.5 lb of salt per gallon is poured into the tank at a rate of 2 gal/min, and the well-stirred mixture leaves the tank at the same rate. How much salt is in the tank after 10 min?

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Problem 7: Find series representations of functions $y_0(x)$, $y_1(x)$ such that the general solution of

$$y'' - xy = 0$$

is $y(x) = c_0 y_0(x) + c_1 y_1(x)$ $(c_0, c_1 \in \mathbb{R})$. Find only enough terms in each series that the pattern of coefficients is obvious; it is not necessary to find a general formula for the n^{th} term in each series.

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Problem 8: Find the general solution $y(t)$ $(t \geq 0)$ of the initial value problem

$$\begin{aligned} y'' + 6y' + 11y &= \delta(t-1) \\ y(0) &= y'(0) = 0 \end{aligned}$$

where δ is the Dirac delta function. Sketch a graph of the solution.

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Problem 9: Consider the periodic function

$$f(x) = x \quad (-2 \le x < 2)$$
$$f(x+4) = f(x)$$

whose graph is shown below. Express $f(x)$ as a Fourier series.