THOMPSON RIVERS UNIVERSITY

MATH 2650
Calculus 3 for Engineering

Instructor: Richard Taylor

MIDTERM EXAM #2
SOLUTIONS

21 Nov 2018 13:00–14:15

Instructions:

1. Read the whole exam before beginning.
2. Make sure you have all 4 pages.
3. Organization and neatness count.
4. Justify your answers.
5. Clearly show your work.
6. You may use the backs of pages for calculations.
7. You may use an approved formula sheet.
8. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		15
2		5
3		5
4		5
5		5
TOTAL:		35

/15

Problem 1: Sketch the region of integration and evaluate:

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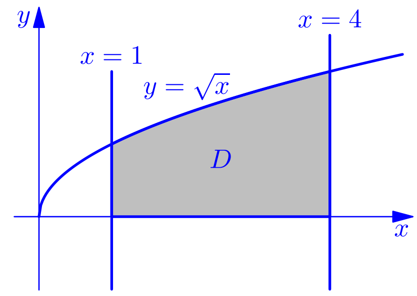
(a) $\displaystyle\iint_D e^{y/\sqrt{x}}\,dA$ where D is bounded by the graph of $y=\sqrt{x}$ and the lines $y=0$, $x=1$ and $x=4$.

$$\begin{aligned}\iint_D e^{y/\sqrt{x}}\,dA &= \int_1^4 \int_0^{\sqrt{x}} e^{y/\sqrt{x}}\,dy\,dx \\ &= \int_1^4 \left[\sqrt{x}e^{y/\sqrt{x}}\right]_{y=0}^{y=\sqrt{x}} dx \\ &= \int_1^4 \left[\sqrt{x}e - \sqrt{x}\right]\,dx \\ &= (e-1)\tfrac{2}{3}x^{3/2}\Big|_1^4 = \boxed{\frac{14}{3}(e-1)}\end{aligned}$$



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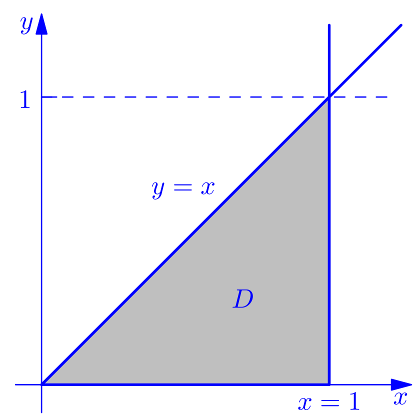
(b) $\displaystyle\int_0^1\int_y^1 x^2e^{xy}\,dx\,dy$

Change the order of integration:

$$\begin{aligned}\iint_D x^2e^{xy}\,dA &= \int_0^1\int_0^x x^2e^{xy}\,dy\,dx \\ &= \int_0^1 \left[xe^{xy}\right]_{y=0}^{y=x} dx \\ &= \int_0^1 \left[xe^{x^2} - x\right]\,dx \\ &= \left[\tfrac{1}{2}e^{x^2} - \tfrac{1}{2}x^2\right]_0^1 \\ &= \left[\tfrac{1}{2}e - \tfrac{1}{2}\right] - \left[\tfrac{1}{2} - 0\right] = \boxed{\tfrac{1}{2}e - 1}\end{aligned}$$



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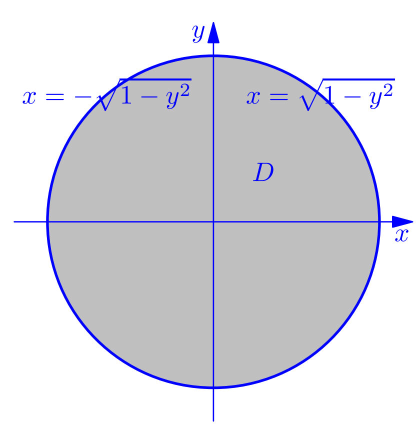
(c) $\displaystyle\int_{-1}^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}(x^2+y^2)\,dx\,dy$

Use polar coordinates:

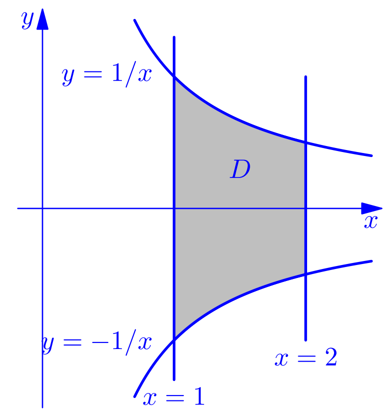
$$\begin{aligned}\iint_D r^2\,dA &= \int_0^{2\pi}\int_0^1 (r^2)(r\,dr\,d\theta) \\ &= \underbrace{\int_0^{2\pi} d\theta}_{2\pi}\underbrace{\int_0^1 r^3\,dr}_{1/4} = \boxed{\frac{\pi}{2}}\end{aligned}$$



/5 **Problem 2:** Calculate the volume of the solid that is bounded by the surfaces $x = 1$, $x = 2$, $y = \pm\frac{1}{x}$, $z = 0$ and $z = x + 1$.

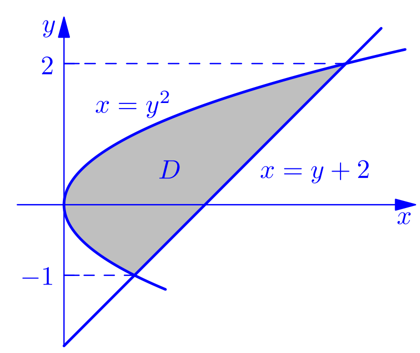
This is just the volume above D (in the xy-plane) and under the graph of $z = x + 1$, so

$$\begin{aligned} V &= \iint_D (x+1)\,dA \\ &= \int_1^2 \int_{-1/x}^{1/x} (x+1)\,dy\,dx \\ &= \int_1^2 \Big[(x+1)y\Big]_{y=-1/x}^{y=1/x} dx \\ &= \int_1^2 (x+1)\frac{2}{x}\,dx \\ &= \int_1^2 \left(2 + \frac{2}{x}\right) dx \\ &= \Big[2x + 2\ln|x|\Big]_1^2 = \big[4 + 2\ln 2\big] - \big[2 + 0\big] = \boxed{2 + 2\ln 2} \end{aligned}$$



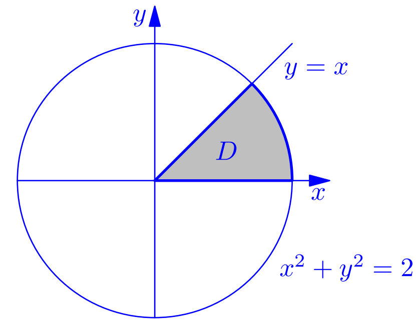
/5 **Problem 3:** Sketch the region whose area is represented by the integral $\int_{-1}^{2}\int_{y^2}^{y+2} dx\,dy$, then evaluate the integral to find the area.

$$\begin{aligned} A = \int_{-1}^{2}\int_{y^2}^{y+2} dx\,dy &= \int_{-1}^{2} x\Big|_{x=y^2}^{x=y+2} dy \\ &= \int_{-1}^{2} (y + 2 - y^2)\,dy \\ &= \Big[\tfrac{1}{2}y^2 + 2y - \tfrac{1}{3}y^3\Big]_{-1}^{2} \\ &= [2 + 4 - \tfrac{8}{3}] - [\tfrac{1}{2} - 2 + \tfrac{1}{3}] = \boxed{\frac{9}{2}} \end{aligned}$$



/5 **Problem 4:** Evaluate $\iint_D xy^2\,dA$ where $D = \{(x,y) : x^2+y^2 \le 2,\ 0 \le y \le x\}$.

In polar coordinates:



$$\begin{aligned}\iint_D xy^2\,dA &= \int_0^{\pi/4}\int_0^{\sqrt{2}} (r\cos\theta)(r\sin\theta)^2(r\,dr\,d\theta)\\ &= \underbrace{\int_0^{\pi/4}\cos\theta\sin^2\theta\,d\theta}_{\frac{1}{3}\sin^3\theta\Big|_0^{\pi/4}}\ \underbrace{\int_0^{\sqrt{2}} r^4\,dr}_{\frac{1}{5}r^5\Big|_0^{\sqrt{2}}}\\ &= \left(\tfrac{1}{3}\sin^3\frac{\pi}{4}\right)\frac{2^{5/2}}{5}\\ &= \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3\frac{4\sqrt{2}}{5} = \boxed{\frac{2}{15}}\end{aligned}$$

/5 **Problem 5:** Evaluate $\int_0^1\int_0^1\int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\,dx\,dz\,dy$

$$\begin{aligned}\int_0^1\int_0^1\int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\,dx\,dz\,dy &= \int_0^1\int_0^1 \left[\frac{zx}{y+1}\right]_{x=0}^{x=\sqrt{1-z^2}} dz\,dy\\ &= \int_0^1\int_0^1 \frac{z\sqrt{1-z^2}}{y+1}\,dz\,dy\\ &= \underbrace{\int_0^1 \frac{1}{y+1}\,dy}_{\ln|y+1|\big|_0^1=\ln 2}\cdot\underbrace{\int_0^1 z\sqrt{1-z^2}\,dz}_{\int_1^0 u^{1/2}(-\frac{1}{2}du)} \qquad (u = 1-z^2,\ du=-2z\,dz)\\ &= \ln 2\cdot\tfrac{1}{3}u^{3/2}\Big|_0^1 = \boxed{\frac{\ln 2}{3}}\end{aligned}$$