Name: ______________________________ Student #: ________________

THOMPSON RIVERS UNIVERSITY

# MATH 211
# Calculus III

Instructor: Richard Taylor

## FINAL EXAM

8 December 2007 09:00–12:00

| PROBLEM | GRADE | OUT OF |
|---|---|---|
| 1 | | 6 |
| 2 | | 5 |
| 3 | | 2 |
| 4 | | 5 |
| 5 | | 6 |
| 6 | | 6 |
| 7 | | 5 |
| 8 | | 8 |
| 9 | | 5 |
| 10 | | 5 |

**Instructions:**

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 8 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

/6

**Problem 1:** Calculate all the first partial derivatives of each of the following functions.

(a) $f(x,y) = y\cos(xy)$

(b) $f(x,y,z) = xye^{y+z}$

/5

**Problem 2:** Consider the function $f(x,y) = \dfrac{xy}{3x-2y}$.

(a) Find an equation of the tangent plane to the graph of $z = f(x,y)$ at $(1,2,-2)$.

/2 **Problem 3:** Write a chain rule for $\dfrac{\partial w}{\partial p}$ given that $w = F(p, q, r, s)$, where $r = f(p, q)$ and $s = g(p, q)$.

/5 **Problem 4:** Let $f(x, y, z) = x^2 + 2y^2 - 3z^2$. Find an equation of the tangent plane to the surface $f(x, y, z) = 3$ at the point $(2, 1, 1)$.

/6 **Problem 5:** Find the maximum value of $x + y + z$ on the ellipsoid $x^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{z}{3}\right)^2 = 1$.

/6

**Problem 6:** Find and classify the critical points of $f(x, y) = xy^2 - x^2y - xy + x^2$.

/5

**Problem 7:** The volume of a cone with radius $R$ and height $h$ can be found by evaluating a double integral of the function $f(r, \theta) = h(1 - \frac{r}{R})$ over an appropriate domain. Set up and evaluate this integral in polar coordinates, and show that it gives the expected result $V = \frac{1}{3}\pi R^2 h$.

/8

**Problem 8:** The temperature (in °C) of a metal sheet at a point $(x, y)$ (with $x$, $y$ in cm) is given by

$$T(x, y) = 100 + 10e^{-x}\sin y.$$

(a) Find the rate of change of temperature at the point $(0, \frac{\pi}{4})$ in the direction of the vector $\mathbf{i} + 3\mathbf{j}$.

(b) Find the direction at $(0, \frac{\pi}{4})$ in which the rate of change of $T$ is greatest, and find this rate of change.

(c) Find the direction(s) at $(0, \frac{\pi}{4})$ in which the directional derivative of $T$ is 0.

(d) An ant located at $(0, \frac{\pi}{4})$ moves with velocity $(-1, 2)\,\text{cm/s}$. What rate of temperature change does the ant experience: (i) in °C/s? (ii) in °C/cm?

/5

**Problem 9:** Evaluate $\iint_D e^{-y^2}\,dA$ where $D$ is the triangular region in the $xy$-plane with vertices $(0,0)$, $(0,1)$ and $(1,1)$.

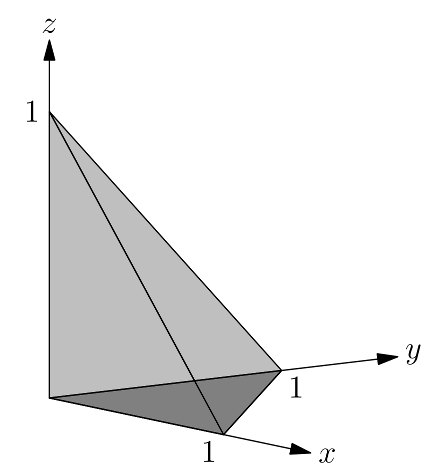
/5

**Problem 10:** For the following iterated integral, sketch the region of integration and evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_x^{\sqrt{x}} \frac{\sin y}{y}\,dy\,dx.$$

/5

**Problem 11:** (a) Set up a double integral that represents the volume of a tetrahedron with vertices at $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and $(0,0,0)$.




(b) Evaluate the integral in part (a).

/5

**Problem 12:** Find an equation of the plane passing through $(1, 1, 1)$ and $(2, 0, 3)$ and perpendicular to the plane $x + 2y - 3z = 0$.

/5

**Problem 13:** Find an equation, in symmetric form, for the line through $(-1, 0, 1)$ that is perpendicular to the plane $2x - y + 7z = 12$.

/5

**Problem 14:** Consider the space curves $\mathbf{r}_1(t) = (t, t^2, 0)$ and $\mathbf{r}_2(s) = (s, s, s^3)$.

(a) Show that the only point of intersection of these curves is at the origin.

(b) Find the angle of intersection between the curves, to the nearest degree.

/8

**Problem 15:** Consider the space curve

$$\mathbf{r}(t) = \sin t \cos t\ \mathbf{i} + \sin^2 t\ \mathbf{j} + \cos t\ \mathbf{k}.$$

(a) Find the unit tangent $\mathbf{T}(t)$.

(b) Find the unit normal $\mathbf{N}(t)$.

(c) Find the curvature $\kappa(t)$.

(d) Find the point(s) of intersection between this curve and the parabolic surface $y = x^2$.