Name: ______________________________ Student #: ________________

THOMPSON RIVERS UNIVERSITY

MATH 2650
Calculus 3 for Engineering

Instructor: Richard Taylor

FINAL EXAM

13 Dec. 2018 09:00–12:00

Instructions:

1. Read the whole exam before beginning.
2. Make sure you have all 11 pages.
3. Organization and neatness count.
4. Justify your answers.
5. Clearly show your work.
6. You may use the backs of pages for calculations.
7. You may use an approved formula sheet.
8. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		4
2		5
3		4
4		4
5		4
6		5
7		7
8		6
9		6
10		9
11		5
12		5
13		7
TOTAL:		71

/4

Problem 1: Let $f(x, y) = \ln\sqrt{x^2 + y^2}$. Show that $f_{xx} + f_{yy} = 0$.

/5

Problem 2: Consider the function

$$f(x, y) = \sqrt{1 + x^2 + y^2 - 2Ax - 2By}$$

where A, B are constants. Find (and simplify) the linear approximation of $f(x, y)$ valid near $(x, y) = (0, 0)$.

/4

Problem 3: Use a chain rule to express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s, if

$$w = x + 2y + z^2, \quad x = \frac{r}{s}, \quad y = r^2 + \ln s, \quad z = 2r.$$

/4

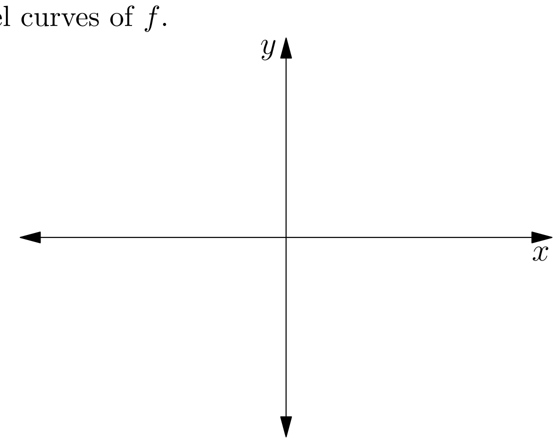
Problem 4: Find a vector $\mathbf{v}$ that is perpendicular to the surface defined by the equation

$$2z^3 = 3(x^2 + y^2)z + \tan^{-1}(xz)$$

at the point $(1, 1, 1)$.

/4 **Problem 5:** Consider the function $f(x, y) = \ln(x^2 + y^2)$.

(a) Sketch a graph of several level curves of f.



/3

(b) On your graph above, indicate the direction of ∇f at the point $(2, 1)$.

/1

/5 **Problem 6:** Find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

/7 **Problem 7:** A bee flies through a cloud in which the temperature [in °C] at any point (x, y, z) is given by the function

$$T(x, y, z) = \frac{x}{y} - yz \qquad (y \neq 0).$$

When the bee is at the point $(4, 1, 1)$:

(a) In what direction should it fly in order to decrease its temperature most rapidly? /2

(b) What is the rate of change of temperature in the direction of fastest decrease? /1

(c) What is the directional derivative of the temperature in the direction of the vector $\mathbf{w} = (1, 1, 0)$? /2

(d) What rate of change of temperature does the bee experience if its velocity is $(1, 0, -2)\,\text{m/s}$? /2

/6

Problem 8: Find all the local maxima, local minima, and saddle points of the function

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy.$$

/6

Problem 9: You are asked to design a cylindrical soup can whose volume is $16\pi\,\text{cm}^3$. What is the least possible surface area that the can can have (including the circles at both ends)? Recall that the volume of a cylinder of radius r and height h is $\pi r^2 h$.

/9 **Problem 10:** Evaluate:

/3 (a) $\iint_D (x^2 + y^2)\, dA$ where D is the triangular region with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

/3 (b) $\int_0^2 \int_x^2 2y^2 \sin(xy)\, dy\, dx$

/3 (c) $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}\, dy\, dx$

/5

Problem 11: Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

/5

Problem 12: Use the substitutions $u = x - y$, $v = 2x + y$ to evaluate the double integral

$$\iint_D x^2 \, dA$$

where D is region bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$, $y = x + 1$.

/7

Problem 13: Evaluate:

/3

(a) $\displaystyle\int_0^1 \int_0^\pi \int_0^\pi y \sin z \, dx \, dy \, dz$

/4

(b) $\displaystyle\iiint_D z \, dV$ where D is the region enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$