# MATH 2110: Quiz #5 – SOLUTIONS

/4 **Problem 1:** Find the unit tangent vector $\mathbf{T}$ at the point where $t = 0$ on the curve described by the the vector function

$$\mathbf{r}(t) = \big(\cos t, 3t, 2\sin 2t\big).$$

$$\mathbf{r}'(t) = \big(-\sin t, 3, 4\cos 2t\big) \implies \mathbf{r}'(0) = \big(0, 3, 4\big)$$

$$\mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{(0,3,4)}{\sqrt{0^2+3^2+4^2}} = \boxed{\frac{1}{5}(0,3,4) = \left(0, \frac{3}{5}, \frac{4}{5}\right)}$$

/6 **Problem 2:** The cylinder $x^2+y^2 = 4$ and the plane $x+y+z = 2$ intersect in a certain curve (actually an ellipse). Write (but do not evaluate) a definite integral whose value gives the length of this curve.

In the $xy$-plane the equation $x^2 + y^2 = 4$ describes a circle, which we can parametrize as

$$x = 2\cos\theta, \quad y = 2\sin\theta \quad (0 \le \theta < 2\pi)$$

To obtain a vector function $\mathbf{r}(\theta)$ for points that also lie on the plane $x + y + z = 2$, we solve for $z$ to obtain

$$z = 2 - x - y = 2 - 2\cos\theta - 2\sin\theta.$$

Thus the cylinder-plane intersection is parametrized as

$$\begin{aligned}\mathbf{r}(\theta) &= (2\cos\theta, 2\sin\theta, 2 - 2\cos\theta - 2\sin\theta)\\ &= 2(\cos\theta, \sin\theta, 1 - \cos\theta - \sin\theta).\end{aligned}$$

This gives

$$\mathbf{r}'(\theta) = 2(-\sin\theta, \cos\theta, \sin\theta - \cos\theta)$$

$$\begin{aligned}\implies |\mathbf{r}'(\theta)| &= 2\sqrt{\sin^2\theta + \cos^2\theta + (\sin\theta - \cos\theta)^2}\\ &= 2\sqrt{1 + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta}\\ &= 2\sqrt{2 - 2\sin\theta\cos\theta}\\ &= \sqrt{8}\sqrt{1 - \sin\theta\cos\theta}.\end{aligned}$$

Therefore the length of the curve is

$$L = \int_0^{2\pi} |\mathbf{r}'(\theta)|\, d\theta = \boxed{\sqrt{8}\int_0^{2\pi} \sqrt{1 - \sin\theta\cos\theta}\, d\theta}$$