MATH 2110: Quiz #4 – SOLUTIONS

/10 **Problem 1:** Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x,y) = 3x + y$ subject to the constraint $x^2 + y^2 = 10$. Sketch a graph of the constraint curve, together with the level curves of f through the points where the extreme values are attained.

Let $g(x,y) = x^2 + y^2$. Then the constraint is $g(x,y) = 10$. Lagrange multipliers gives

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 10 \end{cases} \implies \begin{cases} 3 = \lambda 2x \\ 1 = \lambda 2y \\ x^2 + y^2 = 10 \end{cases} \implies \lambda = \frac{3}{2x} = \frac{1}{2y} \implies x = 3y.$$

$$(3y)^2 + y^2 = 10 \implies 10y^2 = 10 \implies y = \pm 1.$$

Thus the extreme values are attained at the points $(-3,-1)$ and $(3,1)$.

$$\boxed{\begin{array}{l} f(-3,-1) = -10 \text{ is the minimum} \\ f(3,1) = 10 \text{ is the maximum} \end{array}}$$