MATH 2110: Quiz #2 – SOLUTIONS

/3 **Problem 1:** Find the linear approximation $L(x, y)$ for the function $f(x, y) = \dfrac{y-1}{x+1}$ based at the point $(0, 0)$.

Calculate partial derivatives of $f(x, y) = (y-1)(x+1)^{-1}$:

$$f_x = -(y-1)(x+1)^{-2}$$
$$f_y = (x+1)^{-1}$$

Evaluate at $(0, 0)$:

$$f_x(0,0) = 1$$
$$f_y(0,0) = 1$$

Form $L(x, y)$:

$$\begin{aligned} L(x,y) &= f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) \\ &= \boxed{-1 + x + y} \end{aligned}$$

/2 **Problem 2:** Write out the chain rule for $\dfrac{\partial w}{\partial u}$ where $w = f(x, y, z)$, $x = x(u, v)$, $y = y(u, v)$ and $z = z(u, v)$.

$$\boxed{\frac{\partial w}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial u}}$$

/5 **Problem 3:** Find the rate of change of $f(x, y) = x^2 \ln y$ at the point $P(3, 1)$, in the direction of the unit vector $\mathbf{u} = (\frac{3}{5}, \frac{4}{5})$.

$$\nabla f = (f_x, f_y) = (2x \ln y, x^2/y) \implies \nabla f(3,1) = (0, 9)$$

$$\implies D_{\mathbf{u}} f(x,y) = \nabla f \cdot \mathbf{u} = (0,9) \cdot (\tfrac{3}{5}, \tfrac{4}{5}) = \boxed{\frac{36}{5}}$$