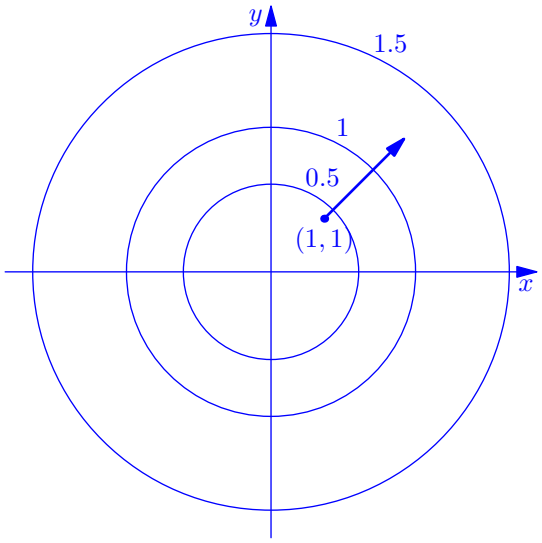
MATH 2110: Quiz #1 – SOLUTIONS

/4 **Problem 1:** Draw a contour map (i.e. the level curves) of the function $f(x,y) = \ln(x^2+y^2)$. At the point $(1,1)$ indicate the direction in which $f(x,y)$ is increasing the fastest.

$$\ln(x^2+y^2) = C \implies x^2+y^2 = e^C \text{ (a circle of radius } e^{C/2})$$



/3 **Problem 2:** Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for the function $f(x,y) = \dfrac{e^{2y}}{x+y^2}$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[e^{2y}(x+y^2)^{-1}\right] = -e^{2y}(x+y^2)^{-2} = \boxed{-\frac{e^{2y}}{(x+y^2)^2}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\frac{e^{2y}}{x+y^2} = \boxed{\frac{2e^{2y}(x+y^2) - e^{2y}(2y)}{(x+y^2)^2}}$$

/3 **Problem 3:** Find all the second partial derivatives of $f(x,y) = \sqrt{x^2+y^2}$.

$$f(x,y) = (x^2+y^2)^{1/2}$$

$$f_x = \tfrac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x = x(x^2+y^2)^{-1/2}$$
$$f_y = \tfrac{1}{2}(x^2+y^2)^{-1/2}\cdot 2y = y(x^2+y^2)^{-1/2}$$

$$\implies f_{xx} = (x^2+y^2)^{-1/2} + x(-\tfrac{1}{2})(x^2+y^2)^{-3/2}\cdot 2x$$
$$= \boxed{(x^2+y^2)^{-1/2} - x^2(x^2+y^2)^{-3/2} = \frac{y^2}{(x^2+y^2)^{3/2}}}$$

$$\implies f_{yy} = (x^2+y^2)^{-1/2} + y(-\tfrac{1}{2})(x^2+y^2)^{-3/2}\cdot 2y$$
$$= \boxed{(x^2+y^2)^{-1/2} - y^2(x^2+y^2)^{-3/2} = \frac{x^2}{(x^2+y^2)^{3/2}}}$$

$$\implies f_{xy} = -\tfrac{1}{2}x(x^2+y^2)^{-3/2}\cdot 2y$$
$$= \boxed{-xy(x^2+y^2)^{-3/2} = -\frac{xy}{(x^2+y^2)^{3/2}}}$$