THOMPSON RIVERS UNIVERSITY

MATH 124
Calculus II

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MIDTERM EXAM #1
SOLUTIONS

16 Feb. 2007 08:30–09:20

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 5 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		18
2		10
3		5
4		3
TOTAL:		36

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Problem 1: Evaluate the following integrals.

(a) $\displaystyle\int_0^{\pi} (1+\cos x)\,dx$

SOLUTION:

$$\int_0^{\pi} (1+\cos x)\,dx = (x+\sin x)\Big|_0^{\pi} = (\pi+0)-(0+0) = \boxed{\pi}$$

(b) $\displaystyle\int x^2 e^{-x}\,dx$

SOLUTION:

integration by parts: $\begin{cases} u = x^2 & dv = e^{-x}\,dx \\ du = 2x\,dx & v = -e^{-x} \end{cases}$

$$\Longrightarrow \int x^2 e^{-x}\,dx = -x^2e^{-x} + \int 2xe^{-x}\,dx$$

integration by parts: $\begin{cases} u = 2x & dv = e^{-x}\,dx \\ du = 2\,dx & v = -e^{-x} \end{cases}$

$$\Longrightarrow \int x^2 e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} + \int 2e^{-x}\,dx$$

$$= \boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$$

$$= \boxed{-(x^2+2x+2)e^{-x} + C}$$

(c) $\displaystyle\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx$

SOLUTION:

substitution: $u = x^2+1 \quad du = 2x\,dx$

$$\Longrightarrow \int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx = \int_1^4 \frac{2\,du}{\sqrt{u}} = 4u^{1/2}\Big|_1^4 = 4\cdot(2-1) = \boxed{4}$$

Problem 1 continued...

(d) $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

SOLUTION:

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx = [\text{area of half-circle of radius 3}] = \tfrac{1}{2}\pi(3)^2 = \boxed{\frac{9\pi}{2}}$$

(e) $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2}\, dx$

SOLUTION:

$$\text{substitution:} \quad u = 1 + \sqrt{x}; \quad du = \frac{1}{2\sqrt{x}}\, dx$$

$$\Longrightarrow \int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2}\, dx = \int \frac{2\, du}{u^2} = -2u^{-1} + C = -2(1 + \sqrt{x})^{-1} + C$$

(f) $\int_{0}^{\infty} \frac{x\, dx}{(x^2 + 4)^{3/2}}$

SOLUTION:

$$\text{substitution:} \quad u = x^2 + 4 \quad du = 2x\, dx$$

$$\begin{aligned} \Longrightarrow \int_{0}^{\infty} \frac{x\, dx}{(x^2 + 4)^{3/2}} &= \int_{4}^{\infty} \frac{1}{2} \frac{du}{u^{3/2}} \\ &= \lim_{t \to \infty} \int_{4}^{t} \tfrac{1}{2} u^{-3/2}\, du \\ &= \lim_{t \to \infty} \Big[-u^{-1/2} \Big]_{4}^{t} = \lim_{t \to \infty} \left[\frac{1}{2} - \frac{1}{\sqrt{t}} \right] = \boxed{\frac{1}{2}} \end{aligned}$$

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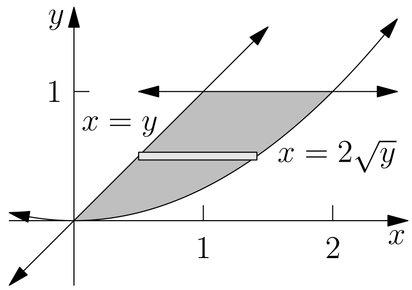
Problem 2: Consider the region bounded by the graphs of $y = x$ and $y = \frac{1}{4}x^2$, to the right of $x = 0$ and below $y = 1$.

(a) Sketch and find the area of the region described above.

SOLUTION:



horizontal strips: $dA = (2\sqrt{y} - y)\,dy$

$$A = \int dA = \int_0^1 (2\sqrt{y} - y)\,dy$$
$$= \left(2 \cdot \tfrac{2}{3}y^{3/2} - \tfrac{1}{2}y^2\right)\Big|_0^1 = \boxed{\frac{5}{6}}$$

(b) The region described is revolved about the x-axis. Sketch and find the volume of revolution.

SOLUTION:

shells: $dV = 2\pi yh\,dy = 2\pi y(2\sqrt{y} - y)\,dy$

$$V = \int dV = \int_0^1 2\pi y(2\sqrt{y} - y)\,dy$$
$$= 2\pi \int_0^1 (2y^{3/2} - y^2)\,dy$$
$$= 2\pi \left[2 \cdot \tfrac{2}{5}y^{5/2} - \tfrac{1}{3}y^3\right]_0^1 = \boxed{\frac{14\pi}{15}}$$

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Problem 3: Find derivatives of the following functions.

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(a) $f(x) = \int_0^x \frac{\sin t}{t}\, dt$

SOLUTION:

$$\boxed{f'(x) = \frac{\sin x}{x}}$$

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(b) $g(x) = \int_{\sqrt{x}}^0 \sin(t^2)\, dt$

SOLUTION:

$$\text{substitute:} \quad u = \sqrt{x}$$

$$\Longrightarrow \; g(x) = \int_u^0 \sin(t^2)\, dt = -\int_0^u \sin(t^2)\, dt$$

$$\text{chain rule:} \quad \frac{dg}{dx} = \frac{dg}{du}\frac{du}{dx} = -\sin(u^2) \cdot \tfrac{1}{2} x^{-1/2} = \boxed{-\frac{\sin x}{2\sqrt{x}}}$$

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Problem 4: A particle moves along a straight line. Its velocity at time t is given by the function $v(t)$. What physical interpretation can be given to the function $f(t) = \int_0^t v(s)\, ds$?

SOLUTION:

$f(t) = \int_0^t v(s)\, ds = \int_0^t x'(s)\, ds = x(t) - x(0) =$ net displacement from time 0 to time t.