Name: ______________________ Student #: ______________

THOMPSON RIVERS UNIVERSITY

MATH 124
Calculus II

S01 – Richard Taylor

FINAL EXAM

18 April 2007 14:00–17:00 Gym M

PROBLEM	GRADE	OUT OF
1		20
2		4
3		2
4		4
5		3
6		7
7		5
8		4
9		6
10		3
11		4
12		9
13		3
14		6
15		5
16		5
TOTAL:		90

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 10 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

/20

Problem 1: Evaluate the following integrals.

(a) $\displaystyle\int \left(10x^4 - \frac{1}{6x^3} + \frac{5}{x} - 3e^2 - \frac{1}{6\sqrt{x}}\right) dx$

(b) $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\, dx$

(c) $\displaystyle\int \frac{x}{\sqrt{x+1}}\, dx$

(d) $\displaystyle\int \sin^3 x \cos^2 x\, dx$

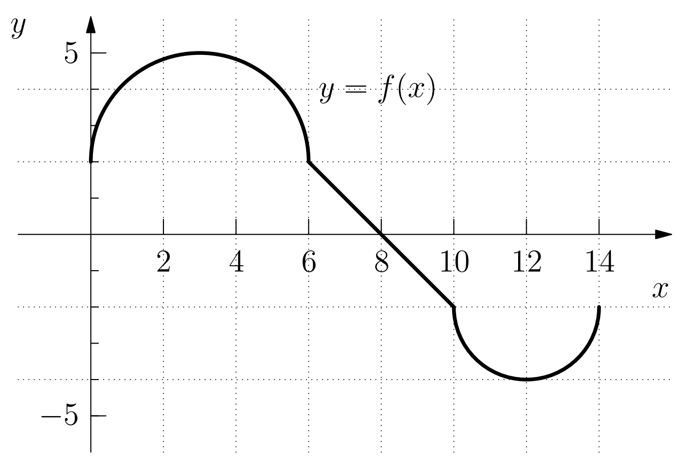
Problem 1 continued...

(e) $\displaystyle\int \frac{2x+1}{(x-3)(x+2)}\,dx$

(f) $\displaystyle\int_0^{\pi/6} x\cos(2x)\,dx$

/4

Problem 2: The following graph of f consists of straight line segments and semicircles.



Use the graph of f to evaluate:

(a) $\displaystyle\int_0^{14} f(x)\,dx$

(b) $\displaystyle\int_{12}^{3} f(x)\,dx$

/2 **Problem 3:** Find $\dfrac{d}{dx}\displaystyle\int_1^{x^2} e^{\sin t}\,dt$.

/4 **Problem 4:** The following table gives velocity data for a bug crawling in a straight line across a sticky surface.

t [seconds]	0	1	2	3	4	5	6
$v(t)$ [mm/s]	1.5	1.2	1.1	1.0	0.9	0.8	0.6

(a) Use the midpoint rule with $n = 3$ rectangles to estimate the distance traveled by the bug during the 6-second time interval.

(b) Use the trapezoid rule with $n = 3$ trapezoids to approximate the distance traveled by the bug during the 6-second time interval.

/3 **Problem 5:** An environmental study in a certain community suggests that the level of carbon monoxide in the air is increasing at a rate $L'(t) = 10/(2t+5)$ parts per million per year, where t is the time in years, and $t = 0$ is the present. If the current level of carbon monoxide is 3.4 parts per million, what will be the level 3 years from now?

/7

Problem 6: For each of the following, determine if the improper integral converges or diverges. If the integral converges, evaluate it.

(a) $\displaystyle\int_2^3 \frac{dx}{x-2}$

(b) $\displaystyle\int_0^\infty xe^{-x}\,dx$

/5

Problem 7: A tank initially contains 100 liters of water in which 50 kg of salt is dissolved. Water containing 2 kg/L salt runs into the tank at the rate of 5 L/min. The mixture is kept thoroughly mixed and flows out of the tank at 5 L/min. Find an expression for the amount of salt (in kg) in the tank t minutes after the process starts.

/4 **Problem 8:** Solve the differential equation $\dfrac{dy}{dx} = \dfrac{\sin x}{y}$ with the initial condition $y\left(\dfrac{\pi}{2}\right) = 2$.

/6 **Problem 9:** Determine (and justify) whether each of the following series is convergent or divergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2+(-1)^n}{5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n!}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\ln n}$

/3

Problem 10: Find the area of the region bounded between the graphs of $y = \sqrt{x}$, $y = x - 2$, $x = 2$, and $x = 5$.

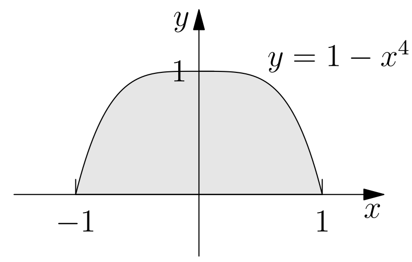
/4

Problem 11: Suppose the temperature (in degrees Celsius) in Kamloops on a certain winter day can be modeled by the formula $T(t) = 2 - \frac{1}{7}(t - 13)^2$, $0 \leq t \leq 24$, where t is the time (in hours) past midnight. Find the average temperature between 2 AM and 2 PM.

/9

Problem 12: Refer to the shaded region under the graph to the right.

(a) Find the area of the shaded region.



(b) Find the volume of the solid formed by revolving the shaded region about the x-axis.

(c) Set up (but *do not evaluate*) an integral for the volume of the solid formed by revolving the same shaded region about the line $x = 2$.

/3

Problem 13: Set up (but *do not evaluate*) an integral for the arc length of the curve $y = \ln(\sin x)$ from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$.

/6

Problem 14: The amount of electricity used per day, in *thousands* of kilowatt-hours, at in industrial plant is a random variable with the probability density function:

$$f(x) = \begin{cases} k(10 - x^2) & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant k.

(b) Find the expected, or mean, electricity usage μ.

(c) Find the probability that the plant will use less then 1000 killowatt-hours on a given day.

/5

Problem 15: (a) Find the first three terms in the Maclaurin series for $f(x) = \ln(1 + 2x)$.

(b) Use your answer to part (a) to estimate $f(0.1)$.

/5

Problem 16: Given that the Maclaurin series for $\arctan x$ is

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

(a) Find the Maclaurin series for $\arctan(x^2)$.

(b) Find the Maclaurin series for $x \arctan x$.

(c) Use the first three non-zero terms of the power series in part (a) to approximate the definite integral

$$\int_0^{1/2} \arctan(x^2)\, dx.$$