Thompson Rivers University

MATH 1240
Calculus II

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MIDTERM EXAM #1
SOLUTIONS

13 February 2013   09:30–10:20

Instructions:

1. Read the whole exam before beginning.
2. Make sure you have all 4 pages.
3. Organization and neatness count.
4. Justify your answers.
5. Clearly show your work.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		16
2		6
3		6
4		6
5		2
TOTAL:		36

/16

Problem 1: Evaluate the following:

/4

(a) $\int x(x^2+3)^{-12/7}\,dx$

$$\begin{cases} u = x^2+3 \\ du = 2x\,dx \end{cases} \implies \int x(x^2+3)^{-12/7}\,dx = \int \frac{1}{2}u^{-12/7}\,du$$
$$= -\frac{1}{2}\cdot\frac{7}{5}u^{-5/7}+C$$
$$= \boxed{-\frac{7}{10}(x^2+3)^{-5/7}+C}$$

/4

(b) $\int_0^{\pi/6} \frac{\sin\theta}{\cos^3\theta}\,d\theta$

$$\begin{cases} u = \cos\theta \\ du = -\sin\theta\,d\theta \end{cases} \implies \int_0^{\pi/6} \frac{\sin\theta}{\cos^3\theta}\,d\theta = \int_1^{\sqrt{3}/2} -\frac{du}{u^3}$$
$$= \frac{1}{2}u^{-2}\Big|_1^{\sqrt{3}/2}$$
$$= \frac{1}{2}\left(\frac{4}{3}-1\right) = \boxed{\frac{1}{6}}$$

/4

(c) $\int_1^2 z^3 \ln z\,dz$

$$\begin{array}{ll} u = \ln z & dv = z^3 \\ du = \frac{1}{z}\,dz & v = \frac{1}{4}z^4 \end{array} \implies \int_1^2 z^3\ln z\,dz = \frac{1}{4}z^4\ln z\Big|_1^2 - \int_1^2 \frac{1}{4}z^3\,dz$$
$$= 4\ln 2 - \frac{1}{16}z^4\Big|_1^2$$
$$= 4\ln 2 - \left(1-\frac{1}{16}\right) = \boxed{4\ln 2 - \frac{15}{16}}$$

/4

(d) $\int \cos(\ln x)\,dx$

$$\begin{array}{ll} u = \cos(\ln x) & dv = dx \\ du = -\frac{1}{x}\sin(\ln x) & v = x \end{array} \implies I = \int \cos(\ln x)\,dx = x\cos\ln x + \int \sin(\ln x)\,dx$$

$$\begin{array}{ll} u = \sin(\ln x) & dv = dx \\ du = \frac{1}{x}\cos(\ln x) & v = x \end{array} \implies I = x\cos(\ln x) + x\sin(\ln x) - \underbrace{\int \cos(\ln x)\,dx}_{I}$$

$$\implies 2I = x\cos(\ln x) + x\sin(\ln x) \implies I = \boxed{\frac{1}{2}\left(x\cos(\ln x)+x\sin(\ln x)\right)}$$

/6

Problem 2: Sketch and find the volume of the solid of revolution formed by revolving the region bounded by the curves

$$y = x^2 \quad \text{and} \quad y = 3x$$

about the x-axis.

intersection point:

$$x^2 = 3x \implies x(x-3) = 0 \implies x = 0, 3$$

volume by washers:

$$dV = \pi(R^2 - r^2)\,dx = \pi[(3x)^2 - (x^2)^2]\,dx$$

$$\implies V = \int_0^3 \pi[(3x)^2 - (x^2)^2]\,dx = \pi \int_0^3 9x^2 - x^4\,dx$$

$$= \pi(3x^3 - \frac{1}{5}x^5)\Big|_0^3 = \boxed{\frac{162}{5}\pi}$$

/6

Problem 3: Sketch and find the area of the region enclosed by the graphs of

$$y = \sqrt{x}, \quad y = 6 - x, \quad \text{and} \quad y = 0.$$

$$y = \sqrt{x} \Leftrightarrow x = y^2 \qquad y = 6 - x \Leftrightarrow x = 6 - y$$

intersection point:

$$y^2 = 6 - y \implies y^2 + y - 6 = (y+3)(y-2) = 0 \implies y = 2, -3$$

area by summing horizontal rectangles:

$$dA = (x_2 - x_1)\,dy = [(6-y) - y^2]\,dy$$

$$\implies A = \int_0^2 [(6-y) - y^2]\,dy = \int_0^2 6 - y - y^2\,dy$$

$$= \Big[6y - \frac{1}{2}y^2 - \frac{1}{3}y^3\Big]_0^2 = \boxed{\frac{22}{3}}$$

/6 **Problem 4:** A biologist innoculates a petri dish with bacterial culture, after which the population of bacteria in the dish is given by the function

$$f(t) = 1000e^{t/5}$$

where t is measured in minutes.

(a) What is the population of bacteria at $t = 10$?

/1

$$f(10) = 1000e^2 \approx 7389$$

(b) What is the population of bacteria at $t = 15$?

/1

$$f(15) = 1000e^3 \approx 20085$$

(c) Between $t = 10\,\text{min}$ and $t = 15\,\text{min}$ what is the *average* population of bacteria?

/4

$$\bar{f} = \frac{1}{15-10}\int_{10}^{15} 1000e^{t/5}\,dt = \frac{1}{5}\cdot 1000 \cdot 5e^{t/5}\Big|_{10}^{15} = 1000(e^3 - e^2) \approx 12696$$

/2 **Problem 5:** Suppose the rate of change of the population of Kamloops is known to be a function $f(t)$, where t is measured in years since 2010. Describe in words what the quantity

$$\int_1^3 f(t)\,dt$$

represents.

It represents the net change in population from 2011 to 2013.