MATH 1230: Quiz #6 – SOLUTIONS

/4 **Problem 1:** Evaluate the infinite series $\sum_{n=1}^{\infty} 2(-e)^{-n}$.

This is a geometric series:

$$\sum_{n=1}^{\infty} 2(-e)^{-n} = 2\sum_{n=1}^{\infty} (\underbrace{-e^{-1}}_{r})^n$$

Since $|r| < 1$ this series converges to

$$2\frac{r}{1-r} = 2\frac{-e^{-1}}{1-(-e^{-1})} = \boxed{\frac{-2e^{-1}}{1+e^{-1}} = \frac{-2}{e+1}}$$

/6 **Problem 2:** Use the ratio test to determine whether the following series converge or diverge:

(a) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

/3 With $a_n = \frac{n^2}{2^n}$ we have:

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2/2^{n+1}}{n^2/2^n} = \frac{1}{2}\left(\frac{n+1}{n}\right)^2$$

so that

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{1}{2}\left(\frac{n+1}{n}\right)^2 = \frac{1}{2}$$

Since $r < 1$, this series **converges**.

(b) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

/3 With $a_n = \frac{(n!)^2}{(2n)!}$ we have:

$$\begin{aligned}\frac{a_{n+1}}{a_n} &= \frac{[(n+1)!]^2/[2(n+1)]!}{(n!)^2/(2n)!} = \left(\frac{(n+1)!}{n!}\right)^2 \frac{(2n)!}{(2n+2)!} \\ &= \left(\frac{1\cdot 2\cdot 3\cdots n(n+1)}{1\cdot 2\cdot 3\cdots n}\right)^2 \frac{1\cdot 2\cdot 3\cdots (2n)}{1\cdot 2\cdot 3\cdots (2n)(2n+1)(2n+2)} \\ &= (n+1)^2\frac{1}{(2n+1)(2n+2)}\end{aligned}$$

so that

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(n+1)^2}{(2n+1)(2n+2)} = \frac{1}{4}.$$

Since $r < 1$, this series **converges**.