MATH 1230: Quiz #5 – SOLUTIONS

/4 **Problem 1:** Evaluate: $\displaystyle\int \sin^2 x \cos^2 x\, dx$

$$\begin{aligned}
\int \sin^2 x \cos^2 x\, dx &= \int (\tfrac{1}{2} - \tfrac{1}{2}\cos 2x)(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x)\, dx \\
&= \int (\tfrac{1}{4} - \tfrac{1}{4}\cos^2 2x)\, dx \\
&= \tfrac{1}{4}x - \tfrac{1}{4}\int \cos^2 2x\, dx \\
&= \tfrac{1}{4}x - \tfrac{1}{4}\int (\tfrac{1}{2} + \tfrac{1}{2}\cos 4x)\, dx \\
&= \tfrac{1}{4}x - \tfrac{1}{4}\big(\tfrac{1}{2}x + \tfrac{1}{8}\sin 4x\big) + C \\
&= \boxed{\tfrac{1}{8}x - \tfrac{1}{32}\sin 4x + C}
\end{aligned}$$

/6 **Problem 2:** Find the function(s) $y(x)$ that satisfy:

(a) $\displaystyle\frac{dy}{dx} = y(x^2+1)$

$$\begin{aligned}
\frac{dy}{y} = (x^2+1)\, dx &\implies \ln|y| = \tfrac{1}{3}x^3 + x + C \\
&\implies y = \pm e^{\frac{1}{3}x^3+x+C} = \underbrace{\pm e^{C}}_{A} e^{\frac{1}{3}x^3+x} \\
&\implies \boxed{y(x) = Ae^{\frac{1}{3}x^3+x}, \quad A \in \mathbb{R}}
\end{aligned}$$

(b) $\displaystyle y' = \frac{x}{y}, \quad y(1) = 2.$

$$\begin{aligned}
y\, dy = x\, dx &\implies \tfrac{1}{2}y^2 = \tfrac{1}{2}x^2 + C \\
&\implies y^2 = x^2 + A \\
&\implies y = \pm\sqrt{x^2+A}, \quad A \in \mathbb{R}.
\end{aligned}$$

$$y(1) = 2 = +\sqrt{1^2 + A} \implies 4 = 1 + A \implies A = 3.$$

$$\implies \boxed{y(x) = \sqrt{x^2+3}}$$