MATH 1230: Quiz #3 – SOLUTIONS

/5 **Problem 1:** Evaluate the following:

(a) $\displaystyle \frac{d}{dx}\int_1^x e^{t^2}\,dt$

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$\boxed{e^{x^2}}$ (by direct application of the Fundamental Theorem)

(b) $\displaystyle f'(x)$ where $\displaystyle f(x)=\int_x^{\sqrt{x}} \ln(w^2)\,dw$

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This also uses the Fundamental Theorem (and chain rule), after some re-arranging:

$$\begin{aligned} f(x) &= \int_x^a \ln(w^2)\,dw + \int_a^{\sqrt{x}} \ln(w^2)\,dw \\ &= -\int_a^x \ln(w^2)\,dw + \int_a^{\sqrt{x}} \ln(w^2)\,dw \end{aligned}$$

$$\begin{aligned} \implies f'(x) &= -\ln(x^2) + \ln(\sqrt{x}^2)\cdot \tfrac{1}{2}x^{-1/2} \\ &= \boxed{-\ln(x^2) + \frac{\ln(x)}{2\sqrt{x}}} \end{aligned}$$

/5 **Problem 2:** Starting with an initial population of 120 people, the population of a small town grows at a rate of $P'(t) = 10 - 2t$ [people per year], where t is measured in years.

(a) What is the population at $t = 6$ years?

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Net change from $t = 0$ to $t = 6$:

$$P(6) - P(0) = \int_0^6 P'(t)\,dt = \int_0^6 (10-2t)\,dt = 10t - t^2\Big|_0^6 = 24$$

$$\implies P(6) = P(0) + 24 = 120 + 24 = \boxed{144 \text{ people}}$$

(b) After how many years does the population reach 0?

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$$P(t) = P(0) + \int_0^6 P'(s)\,ds = 120 + \Big[10s - s^2\Big]_{s=0}^{s=t} = 120 + 10t - t^2$$

So the population reaches 0 when:

$$0 = -t^2 + 10t + 120 \quad \text{(quadratic eq.)}$$

$$\implies t = \frac{-10 \pm \sqrt{10^2 - 4(-1)(120)}}{-2} = 5 \pm \tfrac{1}{2}\sqrt{580}$$

$$\implies t \approx -7.04 \text{ (spurious root) or } \boxed{t \approx 17.04 \text{ years}}$$