MATH 1230: Quiz #2 – SOLUTIONS

/10 **Problem 1:** Evaluate the following:

(a) $\displaystyle\int_0^4 t(t-2)(t-4)\,dt$

$$\int_0^4 (t^3-6t^2+8t)\,dt = \tfrac{1}{4}t^4 - 2t^3 + 4t^2\Big|_0^4$$
$$= \tfrac{1}{4}4^4 - 2\cdot 4^3 + 4\cdot 4^2 = 4^3 - 2\cdot 4^3 + \cdot 4^3 = \boxed{0}$$

(b) $\displaystyle\int_4^9 \frac{2+\sqrt{t}}{\sqrt{t}}\,dt$

$$\int_4^9 (2t^{-1/2}+1)\,dt = 4t^{1/2} + t\Big|_4^9 = 21 - 12 = \boxed{9}$$

Alternate solution:

$$\begin{array}{l} u = 2+t^{1/2} \\ du = \frac{1}{2}t^{-1/2}\,dt \end{array} \implies \int_4^9 \frac{2+\sqrt{t}}{\sqrt{t}}\,dt = \int_4^5 2u\,du = u^2\Big|_4^5 = 25-16 = \boxed{9}$$

(c) $\displaystyle\int x^3(x^4+16)^6\,dx$

$$\begin{array}{l} u = x^4+16 \\ du = 4x^3\,dx \end{array} \implies \int x^3(x^4+16)^6\,dx = \int \tfrac{1}{4}u^6\,du = \tfrac{1}{28}u^7 + C = \boxed{\tfrac{1}{28}(x^4+16)^7 + C}$$

(d) $\displaystyle\int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}\,d\theta$

$$\begin{array}{l} u = \cos\theta \\ du = -\sin\theta\,d\theta \end{array} \implies \int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}\,d\theta = \int_1^{1/\sqrt{2}} \frac{-du}{u^3} = \int_1^{1/\sqrt{2}} (-u^{-3})\,du = \tfrac{1}{2}u^{-2}\Big|_1^{1/\sqrt{2}} = 1 - \tfrac{1}{2} = \boxed{\frac{1}{2}}$$