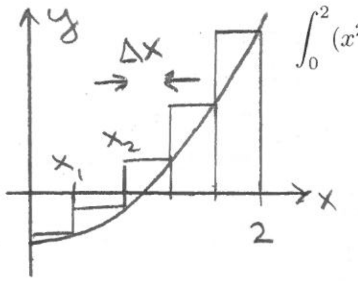
MATH 1230: Quiz #1 – SOLUTIONS

/4 **Problem 1:** Approximate $\int_0^2 (x^2-1)\,dx$ using a Riemann sum of 5 rectangles with heights based at:

(a) the right endpoint of each subinterval. Include a sketch in your solution.

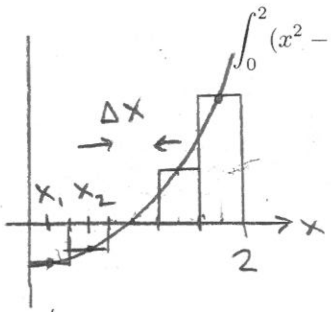
/2 With $f(x) = x^2 - 1$ and $\Delta x = \frac{2}{5}$:



$$\int_0^2 (x^2-1)\,dx \approx \sum_{i=1}^{5} f(x_i)\Delta x = \left[f(\tfrac{2}{5}) + f(\tfrac{4}{5}) + f(\tfrac{6}{5}) + f(\tfrac{8}{5}) + f(\tfrac{10}{5})\right]\tfrac{2}{5}$$
$$= [(-0.84) + (-0.36) + (0.44) + (1.56) + 3]\,\tfrac{2}{5} = \boxed{1.52}$$

(b) the midpoint of each subinterval. Include a sketch in your solution.

/2



$$\int_0^2 (x^2-1)\,dx \approx \sum_{i=1}^{5} f(x_i)\Delta x = \left[f(\tfrac{1}{5}) + f(\tfrac{3}{5}) + f(\tfrac{5}{5}) + f(\tfrac{7}{5}) + f(\tfrac{9}{5})\right]\tfrac{2}{5}$$
$$= [(-0.96) + (-0.64) + (0) + (0.44) + (0.96) + (2.24)]\,\tfrac{2}{5} = \boxed{0.64}$$

/6 **Problem 2:** Evaluate $\int_0^2 x^2 - 1\,dx$ using the definition of the definite integral (i.e. a limit of Riemann sums). The following formulas might be useful: The following formulas might be useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\Delta x = \frac{2}{n}, \quad x_i = i\Delta x = \frac{2i}{n}$$

area of n rectangles:
$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\frac{2}{n}$$
$$= \sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 - 1\right]\frac{2}{n}$$
$$= \frac{8}{n^3}\sum_{i=1}^{n} i^2 - \frac{2}{n}\sum_{i=1}^{n} 1$$
$$= \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} - \frac{2}{n}n$$
$$= \frac{4}{n^2}\,\frac{(n+1)(2n+1)}{3} - 2$$

$$\Longrightarrow \int_0^2 x^2 - 1\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\left[\frac{4}{n^2}\,\frac{(n+1)(2n+1)}{3} - 2\right] = \frac{8}{3} - 2 = \boxed{\frac{2}{3}}$$