THOMPSON RIVERS UNIVERSITY

Department of Mathematics & Statistics

MATH 1230
Calculus 2 for Engineering

Section 02

Instructor: Richard Taylor

FINAL EXAM
(take-home)

21 April 2020   14:00–16:00

Instructions:

1. You have 4 hours to complete and submit the exam.

2. Write your solutions either on your own paper or on a printed copy of this exam. Scan or photograph your solutions (scan to a single PDF document is strongly preferred) and email them to `rtaylor@tru.ca` by 21 April 6:00PM PST. Late submissions will not be accepted.

3. Organization and neatness count.

4. Include the following signed and dated Declaration of Academic Integrity on the first page of your submission (feel free to just print and sign this page and include it with your submission):

   By submitting this work for assessment I hereby declare that it is the result of my own effort and that I did not copy (in whole or in part) the work of any other individual.

   I also declare that subsequent to receiving the assigned work I did not discuss the questions or possible answers with any other person, either face to face or electronically.

   By submitting this declaration I agree to any reasonable level of scrutiny deemed necessary to determine whether I have violated TRU Policy on Student Academic Integrity ED 5-0.

Name: _____   **Signature:** _____

Student #: _____   **Date:** _____

Problem 1: Evaluate the following integrals.

/21

/5   (a) $\int \left(5x^3 - \dfrac{1}{3x^2} + \dfrac{2}{x} - 3e^\pi - 8\sqrt{x} \right) dx$

/4   (b) $\int_0^4 3x\sqrt{25 - x^2}\, dx$

/4   (c) $\int_0^\pi \sin x \cos^2 x \, dx$

Problem 1 continued...

/4 (e) $\displaystyle\int \frac{3x-5}{(x+1)(x+2)}\,dx$

/4 (f) $\displaystyle\int xe^{2x}\,dx$

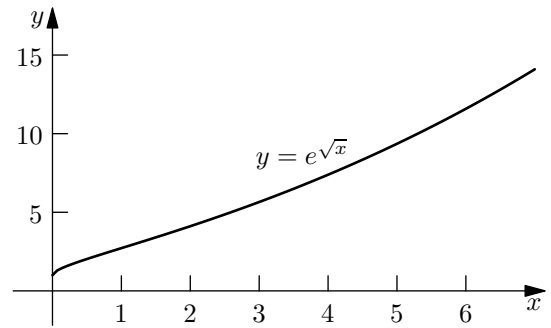/2 **Problem 2:** Find $\displaystyle\frac{d}{dx}\int_1^{x^2}\cos(t^3)\,dt$.

/7 **Problem 3:** Consider the definite integral $\int_0^6 e^{\sqrt{x}}\, dx$.

(a) Use the trapezoid rule with $n = 3$ trapezoids to approximate the integral. Draw the trapezoids on the graph.

/4

(b) Use Simpson's rule with $n = 4$ to approximate the integral.

/3

/4 **Problem 4:** A population study suggests that Earth's human population P is growing at a rate

$$\frac{dP}{dt} = 0.2t - 0.01t^2$$

measured in billions of people per year, with time t measured in years from the present date. If the current population is 7.0 billion, what will be the population 5 years from now?

/6 **Problem 5:** For each of the following, determine if the improper integral converges or diverges. If the integral converges, evaluate it.

/3   (a) $\displaystyle\int_2^3 \frac{dx}{\sqrt{x-2}}$

/3   (b) $\displaystyle\int_1^\infty \frac{1}{x^{3/2}}\, dx$

/5 **Problem 6:** A tank initially contains 500 liters of water in which 20 kg of salt is dissolved. Water containing 1 kg/L of salt flows into the tank at the rate of 4 L/min. The mixture is kept thoroughly mixed and flows out of the tank at 4 L/min. Find an expression for the amount of salt (in kg) in the tank t minutes after the process starts.

/4 **Problem 7:** Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x^2}{y}$ with the initial condition $y(0) = 2$.

/9 **Problem 8:** For each of the following infinite series, determine whether the series is convergent or divergent, and evaluate the sum if possible.
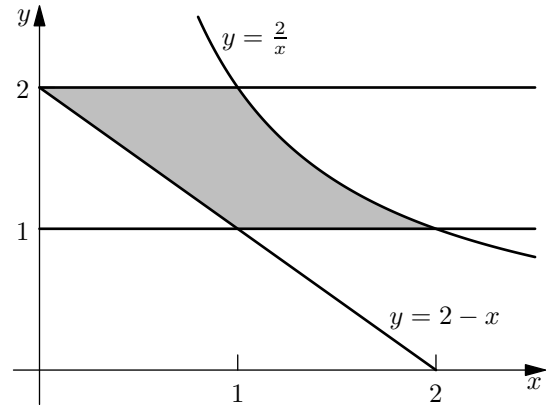
/3 (a) $\displaystyle\sum_{k=0}^{\infty} 2\left(\frac{3}{4}\right)^k$

/3 (b) $\displaystyle\sum_{k=0}^{\infty} 2\left(\frac{4}{3}\right)^k$

/3 (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(2n)!}$

/4 **Problem 9:** Find the area of the region bounded between the graphs of $y = 2 - x$, $y = \dfrac{2}{x}$, $y = 1$, and $y = 2$.


/4 **Problem 10:** During a 4-hour drive from Kamloops to Vancouver, Sarah's speed v (measured in km/h) as a function of time t (measured in hours from the start of her trip) was

$$v(t) = 80 + 40\sin(\pi t/4).$$
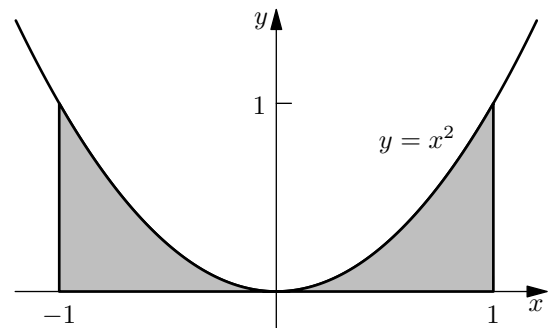
Calculate her average speed over the entire trip.

/4 **Problem 11:** Consider the region bounded by the graphs of $y = 1 - x$, $x = 0$ and $y = 0$. Find the volume of the solid formed by revolving this region about the y-axis.
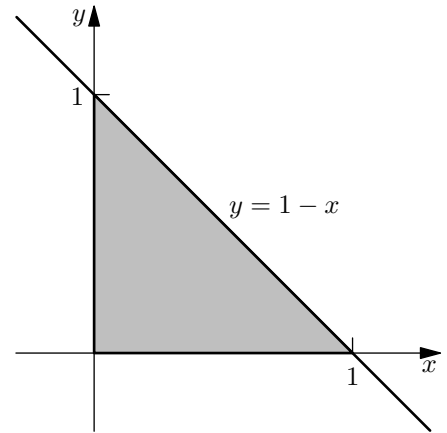
/4 **Problem 12:** Consider the region bounded by the graph of $y = x^2$ and the x-axis, between $x = -1$ and $x = 1$. Find the volume of the solid formed by revolving this region about the x-axis.

/5   **Problem 13:** Consider the function $f(x) = \ln(1 + x)$.

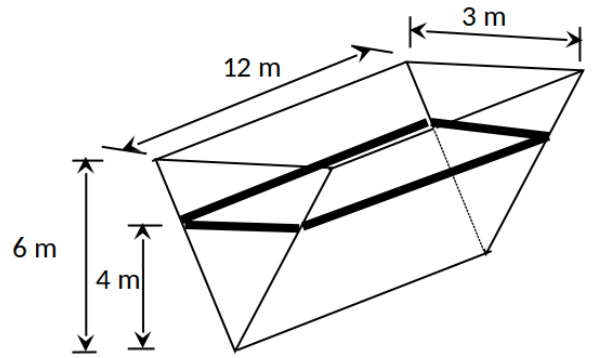/3   (a) Find the first three terms in the Maclaurin series for $f(x)$.

(b) Use your answer to part (a) to estimate $f(0.2)$.

/2

/2   **Problem 14:** Set up (but **do not evaluate**) a definite integral that gives the arc length of the graph

/3   of $y = \sin x$ from $x = 0$ to $x = \dfrac{\pi}{2}$.

/5

Problem 15: The tank shown contains water to a depth of 4 meters. Find the work required to pump all the water out to the level of the top of the tank. Express your answer in terms of the density of water, ρ, and the acceleration of gravity, g.

/8 **Problem 16:** Given that the Maclaurin series for $\sin x$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

/2 (a) Find the Maclaurin series for $\sin(x^2)$.

/2 (b) Find the Maclaurin series for $x \sin x$.

/4 (c) Use the first three non-zero terms of the power series in part (a) to approximate the definite integral

$$\int_0^{1/2} \sin(x^2) \, dx.$$